# Tutorial: The Ouroboros Protocol Family

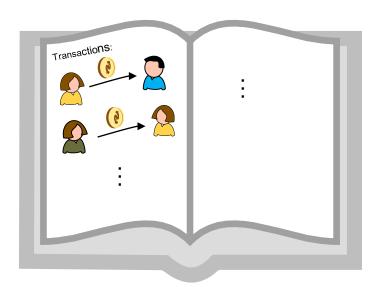
Advances in Financial Technologies, 2019

Christian Badertscher & Aggelos Kiayias

The University of Edinburgh & IOHK

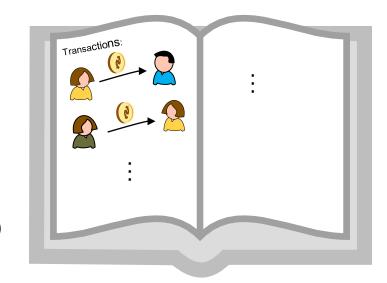
## The Goal of Blockchain Protocols

Implement an immutable transaction ledger...



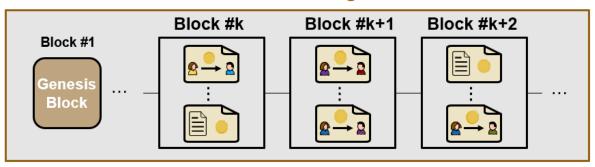
# **Immutable Ledger Properties**

- Submit transactions
- Get included into the ledger (if valid)
- Everyone can access ledger
- Ledger can't be changed (immutable)



# **The Ledger Functionality**

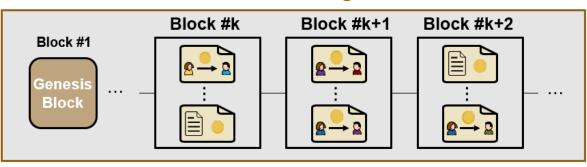
#### A Public Ledger



# The Ledger Functionality

- The functionality formalizes the relevant blockchain properties and the limited capabilities of an adversary.
  - E.g., common state, well-formed blocks, recent transactions etc.

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- The functionality formalizes the relevant blockchain properties and the limited capabilities of an adversary.
  - E.g., common state, well-formed blocks, recent transactions etc.
- Important: It captures the service provided to any cryptographic protocol.

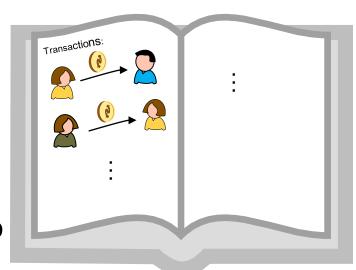
#### **Applications: Incentive Mechanisms, Poker, general MPC**

# Block #k Block #k Block #k+1 Block #k+2 Genesis Block # Genesis Block # Block #k+2 Genesis Block # Bl

Implement an immutable transaction ledger...

#### **But:**

- Avoid a central trusted entity
- Allow dynamic and easy participation
- Be permissionless and accessible to anyone (read and write)



#### The case of Bitcoin:

 Parties repeatedly try to solve cryptographic puzzles. A solution allows to create a block and append it to the chain.

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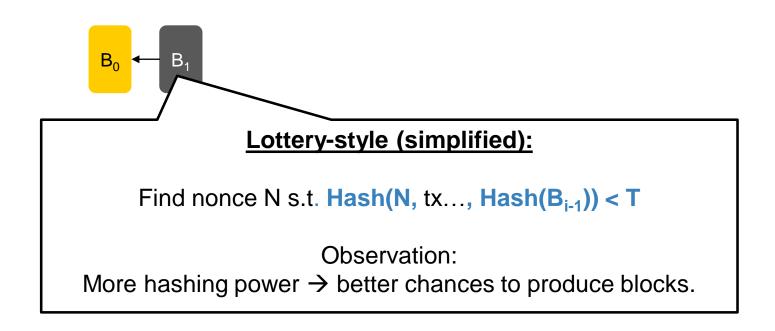
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Genesis Block

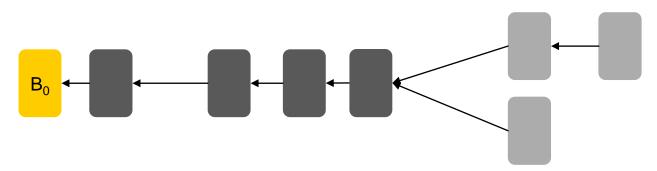
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A Tree Structure (Forks)

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#### **Blockchain Properties [GKL15,PSS17]:**

Common-prefix (CP): Honest miners share a consistent common prefix.

**Chain-growth (CG):** The number of blocks increases over time.

Chain-quality (CQ): A guaranteed fraction of honestly contributed blocks.

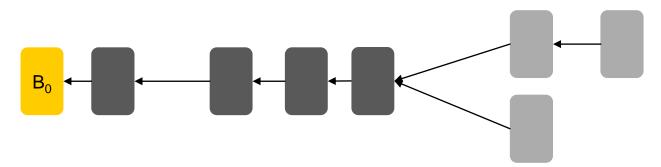
→ Ledger can be realized assuming honest majority of hashing power

A very nice blockchain feature: **Dynamic availability (DA).** 

- Parties join and leave at will. They need to bootstrap a chain when (re-) joining.
  - → Easy in Bitcoin: "longest-chain rule" (general: most difficult chain).
- Number of online/offline parties changes over time
  - → Analysis must account for that.
- No *a priori* knowledge of participation levels is required by the protocol.
- Unannounced disappearance.

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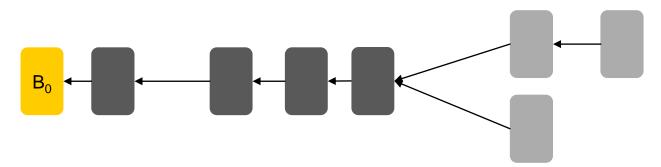
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**Proof-of-Stake to the rescue!** 

#### **Proof-of-Stake Blockchains:**

- Use stake (a virtual resource) instead of hashing power.
- Miners = Stakeholders.
- Next stakeholder to produce block elected with probability proportional to stake.

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#### Two categories:

Nakamoto-style consensus (e.g., Ouroboros, Snow White)

BFT-style consensus (e.g., Algorand, Casper, Ouroboros-BFT)

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#### **Complications of PoS vs. PoW:**

- PoS has costless simulation:
  - No physical resources to create blocks: several transaction histories could be generated "in the adversaries head".
- Long-Range attacks in the threat model:
  Adversary tries to deceive (new) participants into believing the "wrong" history (which are cheap to generate).

#### **Tutorial Overview**

- The development steps of a pure PoS-based blockchain protocol in the dynamic availability setting.
  - Security follows from the "honest majority of stake" assumption.

 Start with the initial version and refine it until all the security requirements are achieved.

#### **Tutorial Overview – Main Content**

Ouroboros "Classic" (Crypto 17) Semi-adaptive adversaries, synchrony Strong mathematical framework

Ouroboros
Praos
(Eurocrypt 2018)

Ouroboros Genesis (CCS 2018)

Ouroboros
Chronos
(In submission, 2019)

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- + Adaptive Adversaries
- + Network Delay ("semi-synchronous")

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- + Full dynamic availability
- + Bootstrapping from Genesis

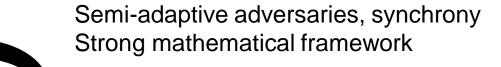
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- + Only based on same-speed assumption.
- + Bootstrapping state and time from genesis

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  - = PoS blockchain in the DA setting without global clocks.



#### **Tutorial Overview – Additional Features**

Ouroboros "Classic" (Crypto 17)



**Ouroboros BFT** 

Ouroboros
Praos
(Eurocrypt 2018)

An extremely simple BFT protocol that follows from the Ouroboros Classic analysis

Ouroboros Genesis (CCS 2018)



Ouroboros Crypsinous (S&P 2019)

Ouroboros
Chronos
(In submission, 2019)

Ouroboros with Privacy

Ouroboros "Classic" (Crypto 17)

Ouroboros
Praos
(Eurocrypt 2018)

Ouroboros Genesis (CCS 2018)

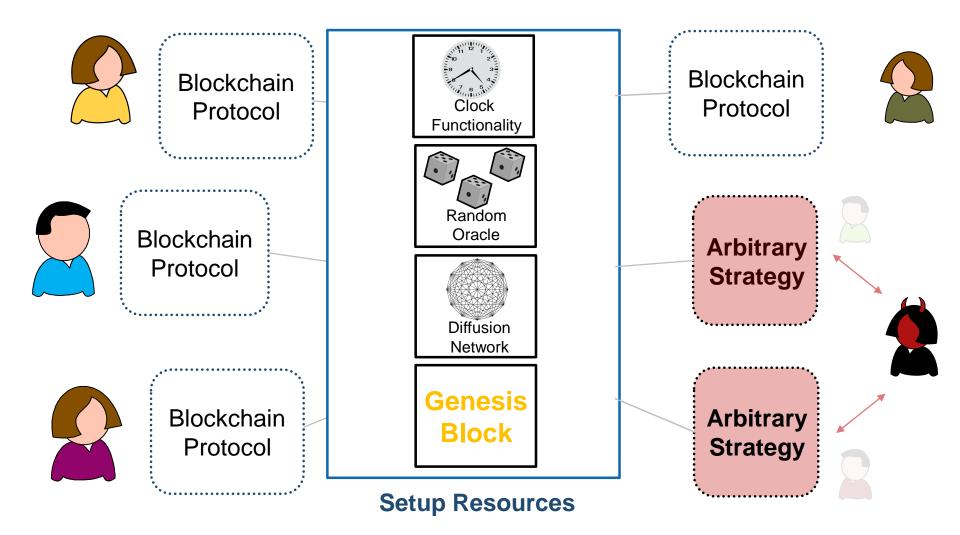
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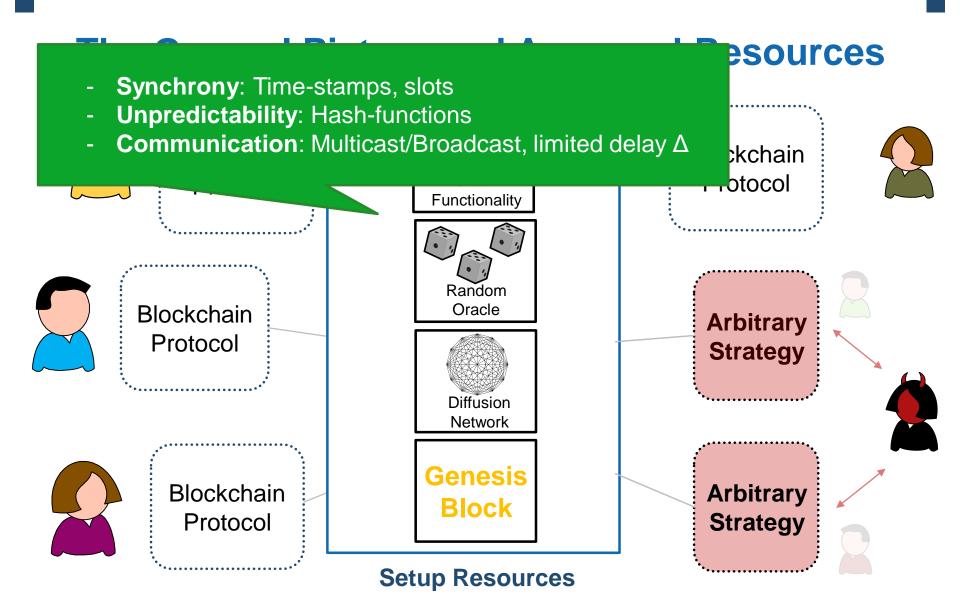
Semi-adaptive adversaries, synchrony Strong mathematical framework

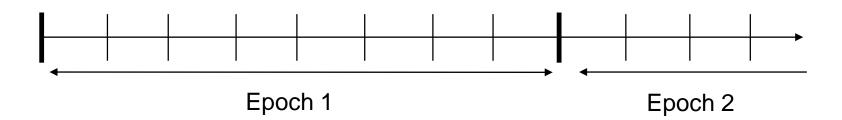
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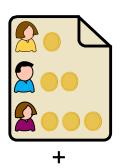


# **The General Picture and Assumed Resources**











2/2 = Public address: verification key vk<sub>i</sub> of a signature scheme

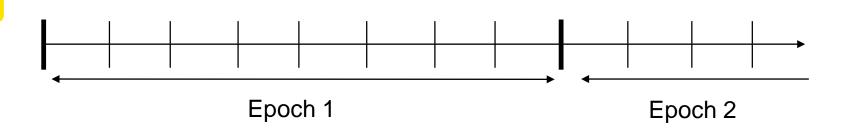


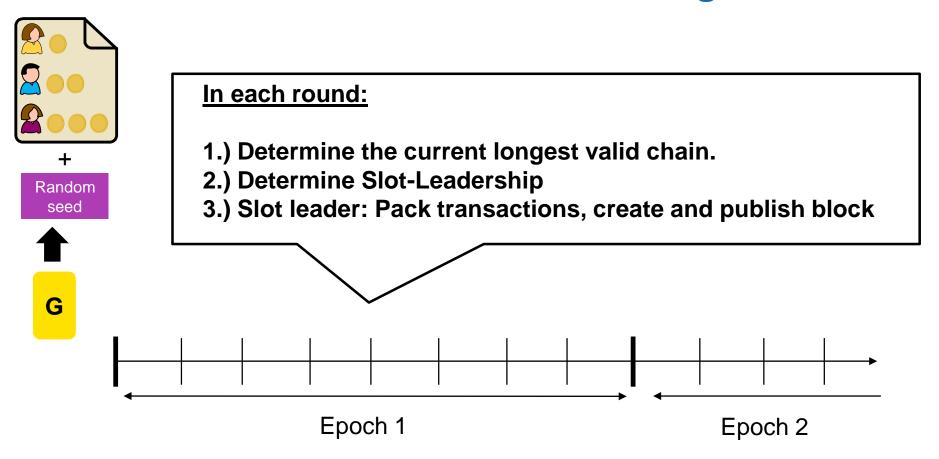
= A number of coins (tokens) s<sub>i</sub> associated to vk<sub>i</sub>

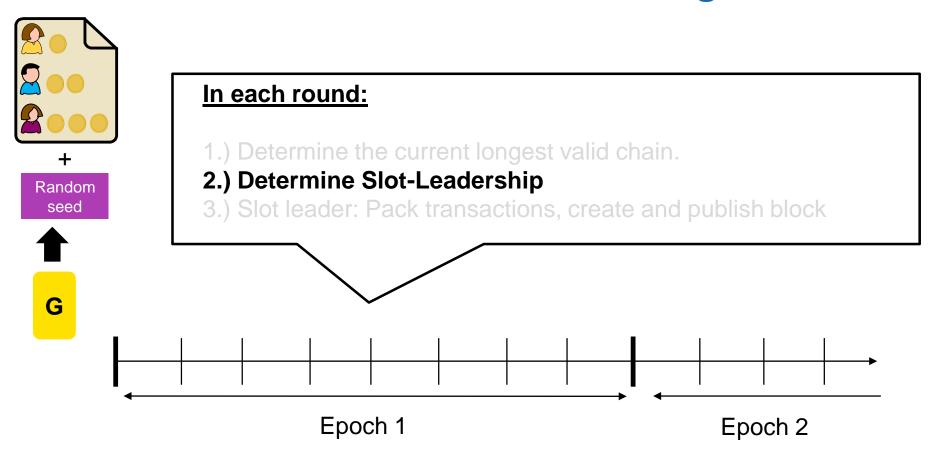












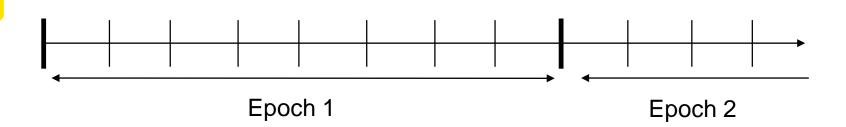


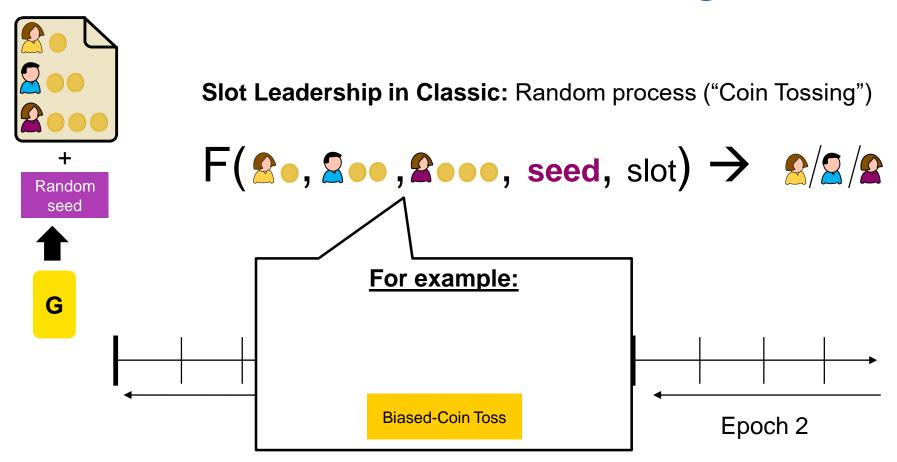
Random seed

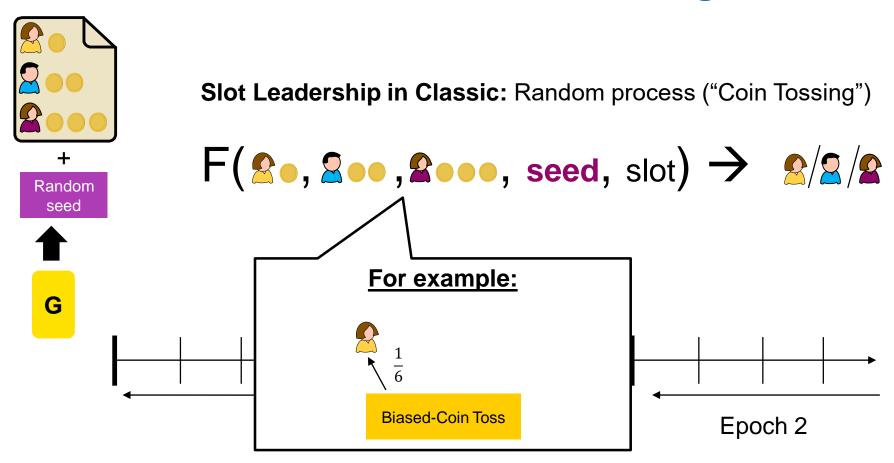


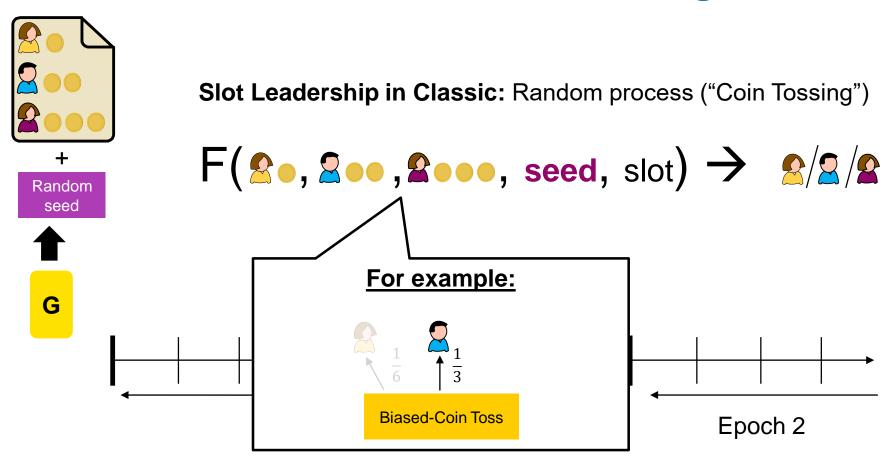


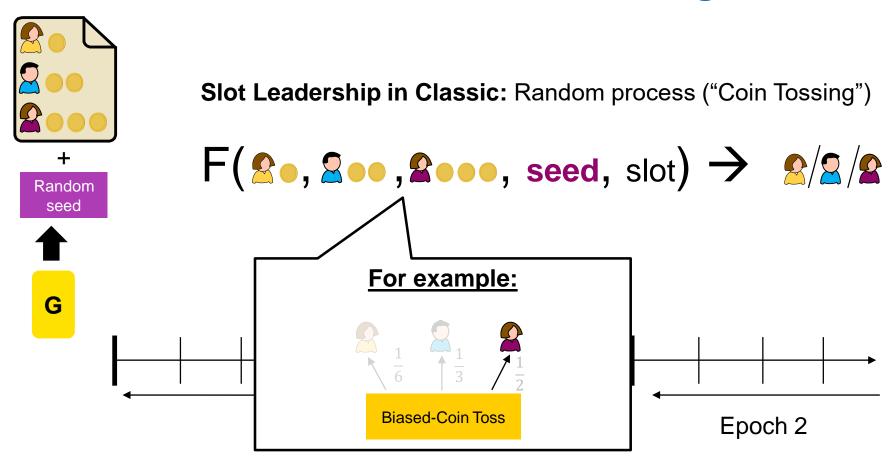
Slot Leadership in Classic: Random process ("Coin Tossing")













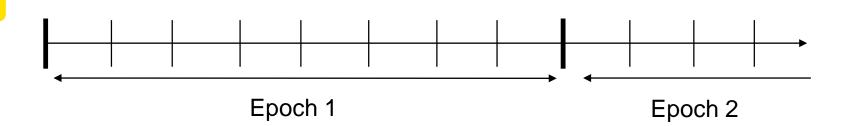
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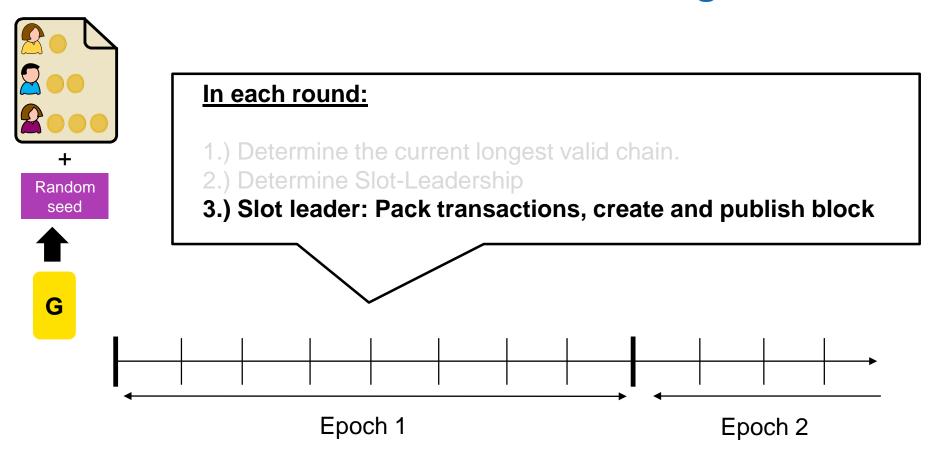
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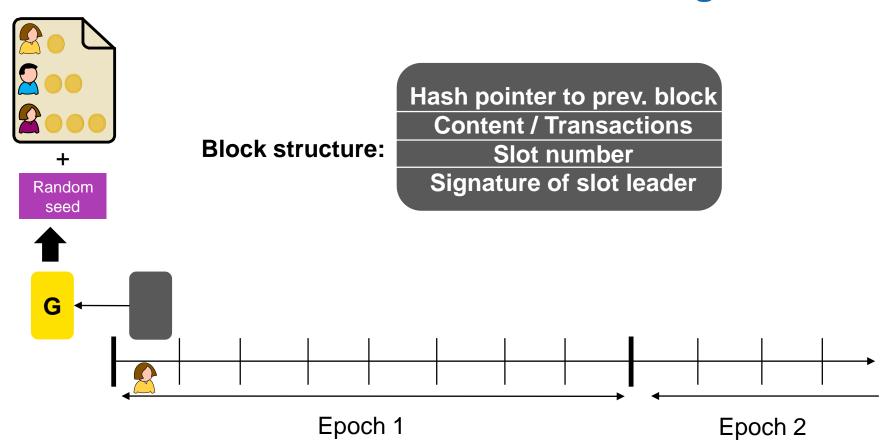
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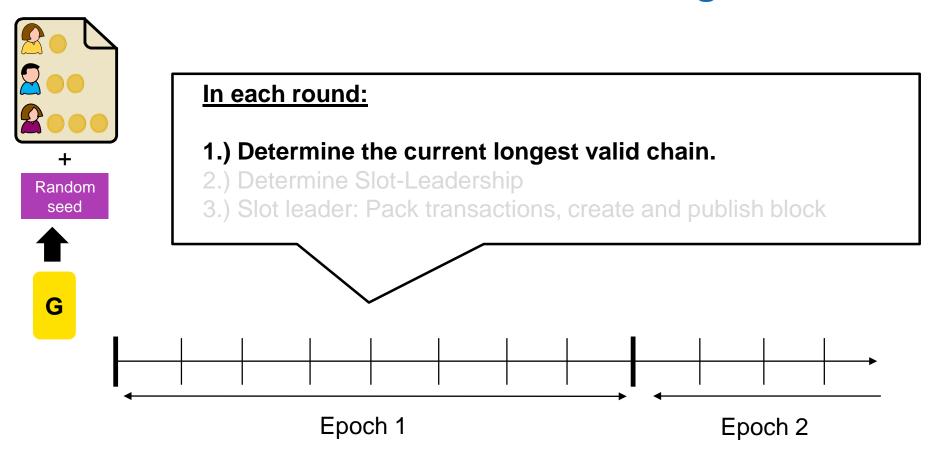


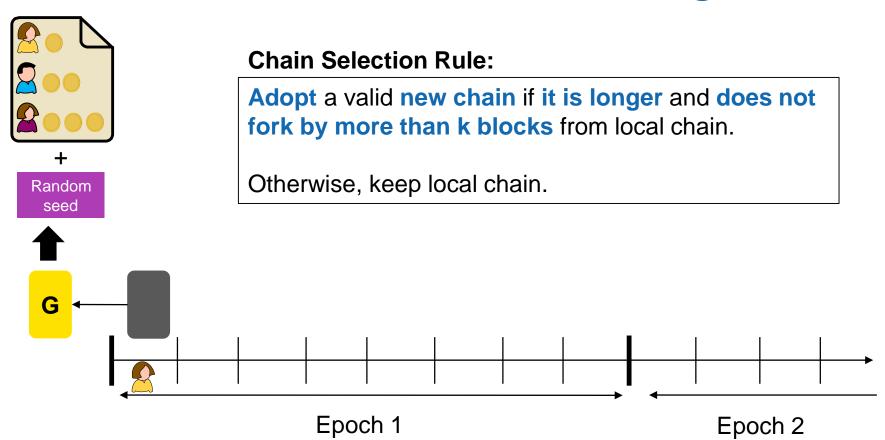
#### Simplified model "Semi-adaptive" (will be strengthened later):

Adversary cannot adaptively react on the (public) slot-leader schedule.
 (As an approximation: think of a static corrupted set of parties)

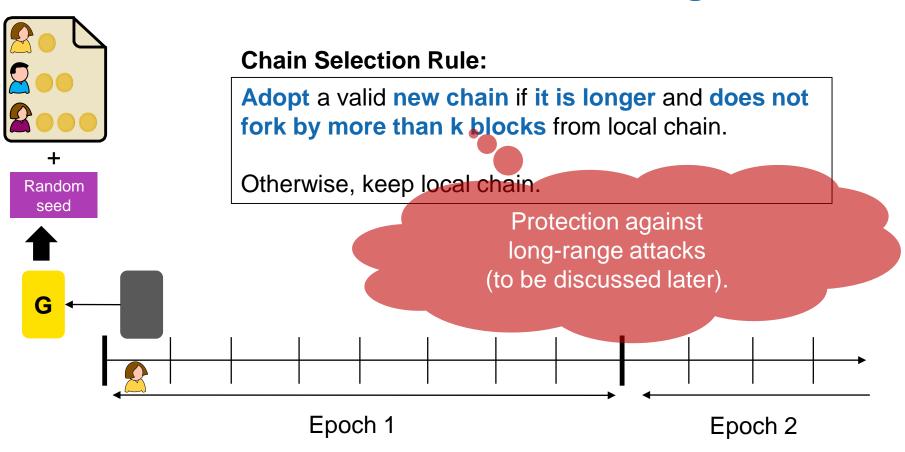




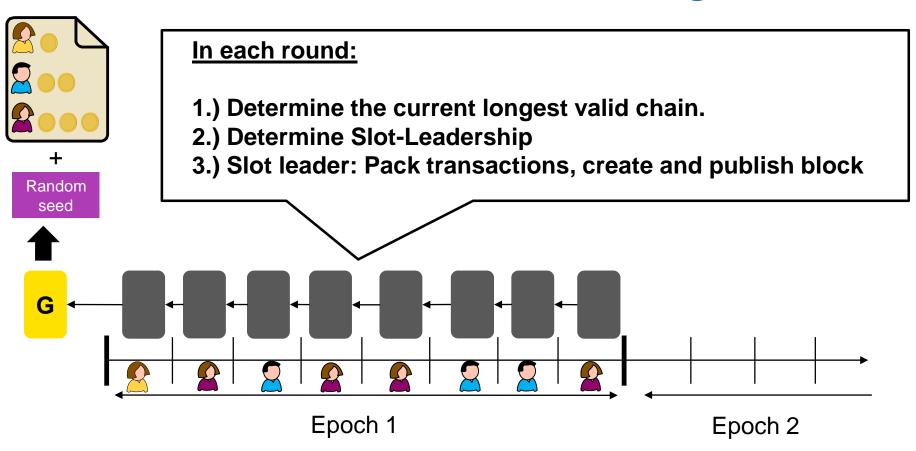


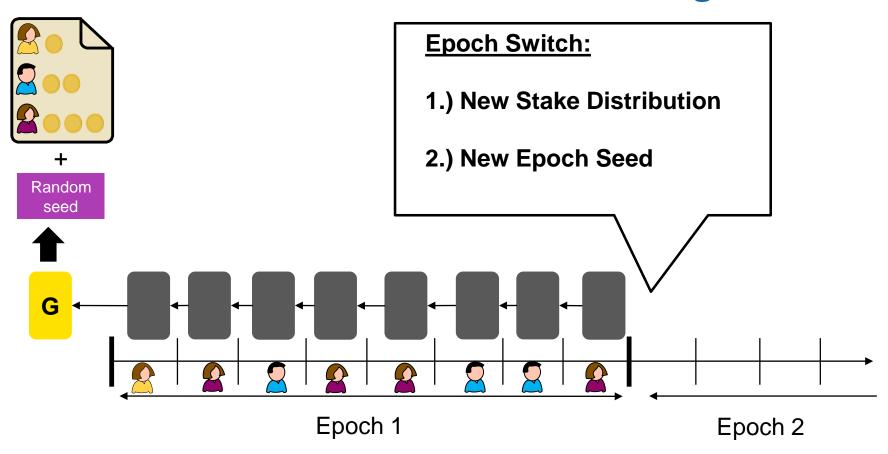


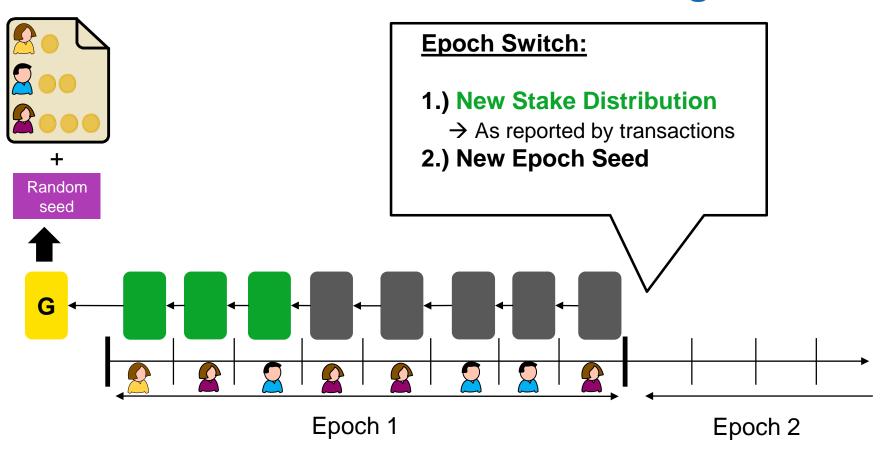
Simplified model: no newcomers, full participation (will be strengthened later).

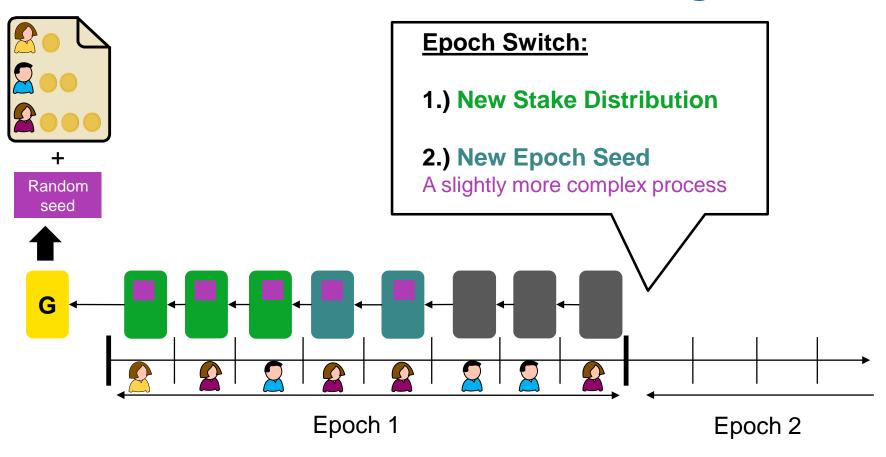


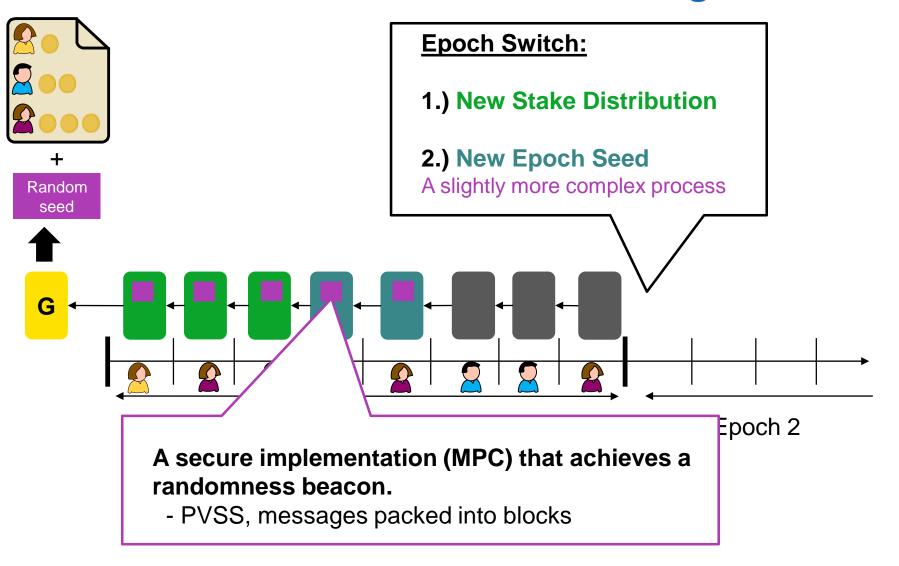
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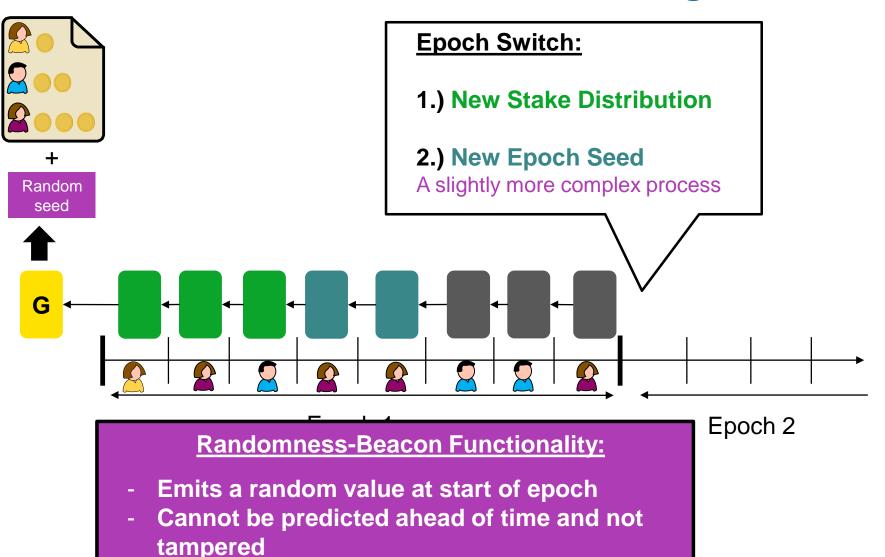


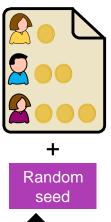








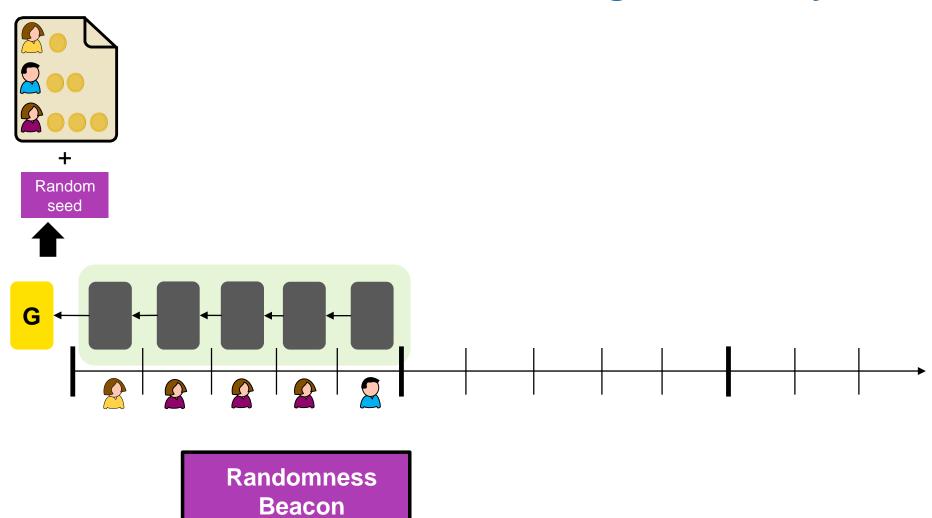


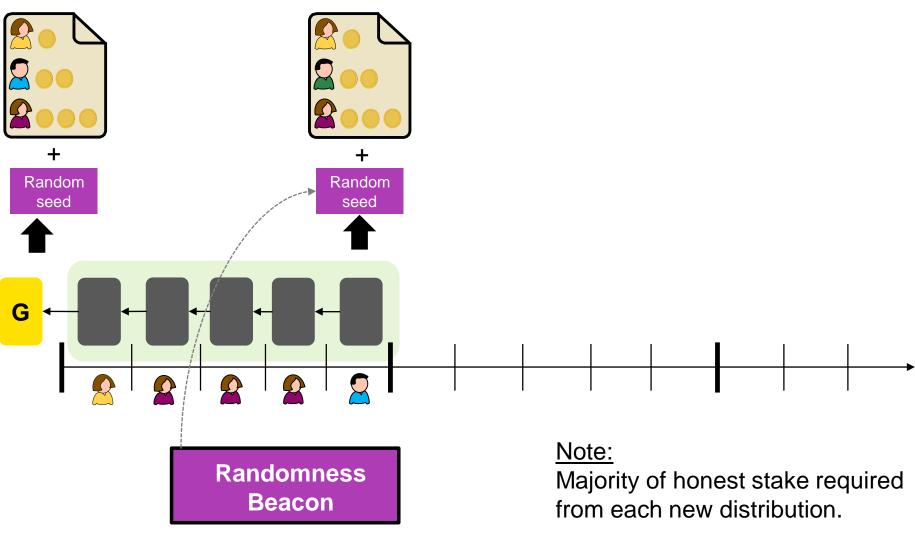


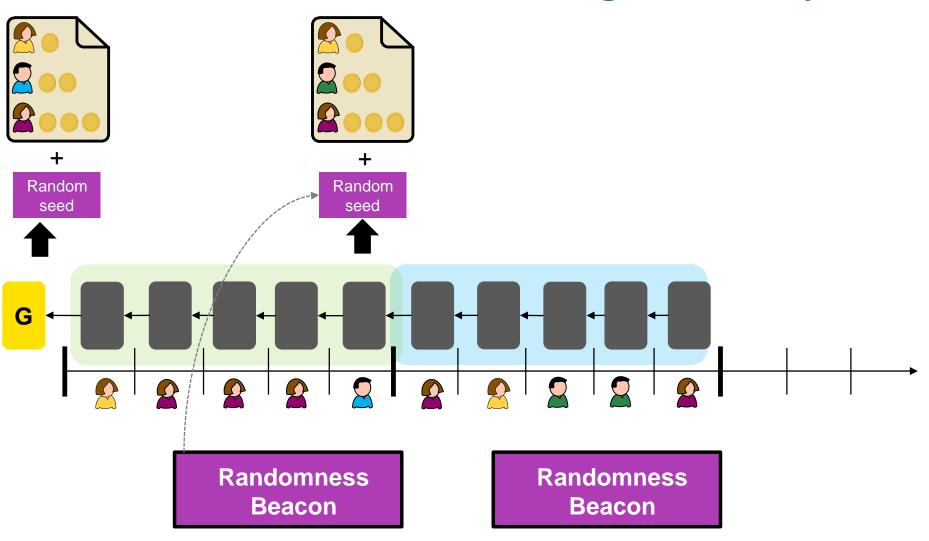


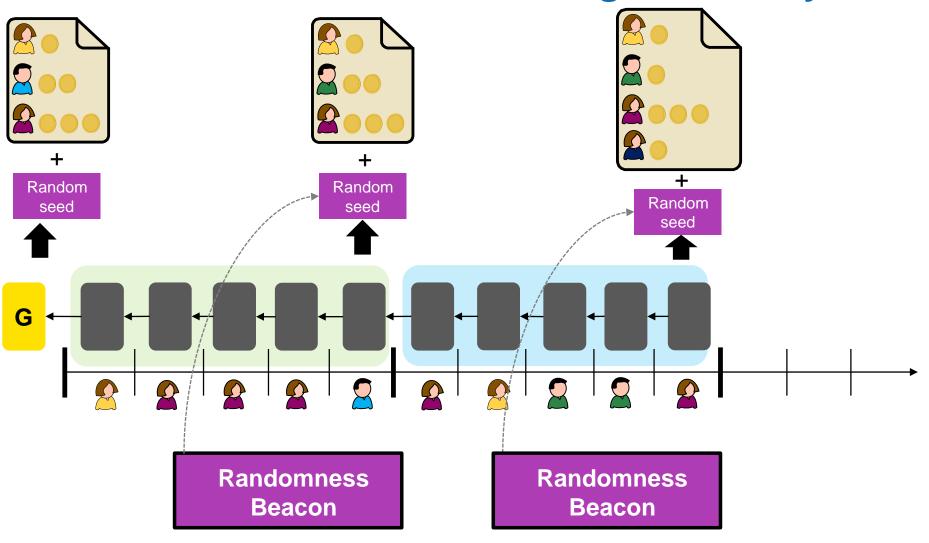


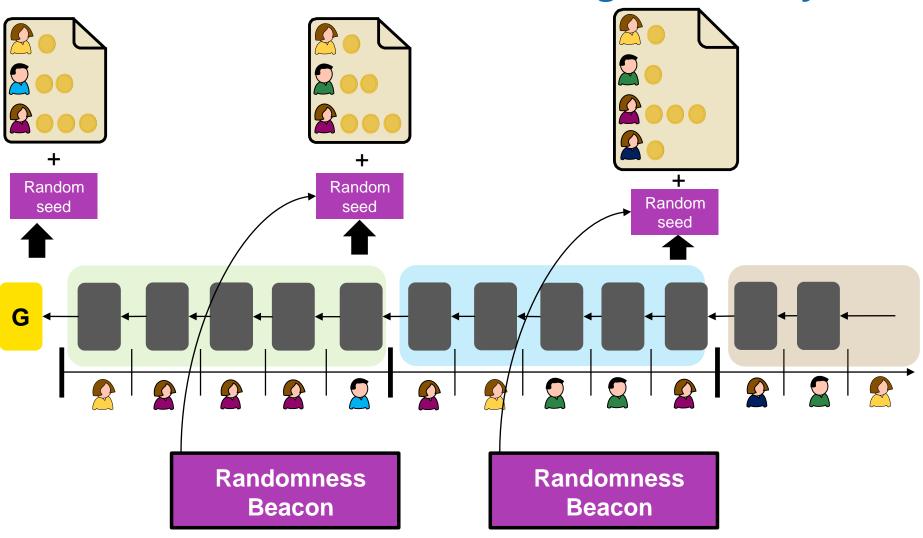










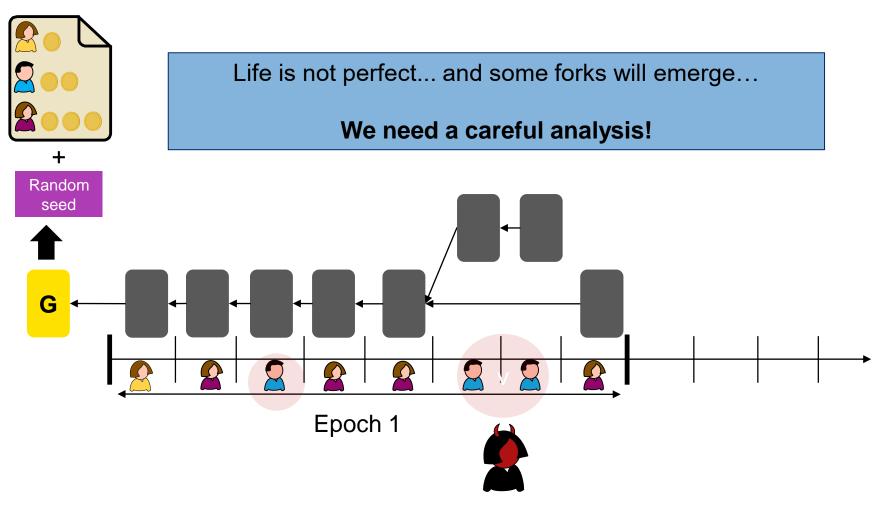


## **Ouroboros – Analysis**

Analysis of first epoch

Lifting to multiple epochs (inductive argument)

## **Ouroboros – Analysis of First Epoch**



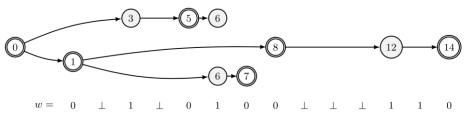
# A General Analytical Approach: The Forkable String Analysis

Slots are assigned a symbol from an alphabet. The symbol signifies whether **honest** parties speak, **adversaries** speak or **no-one** speaks.

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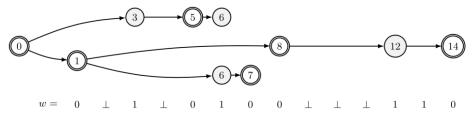
Such a string gives rise to a family of admissible graphs that describe all that can happen in an execution that follows longest chain:



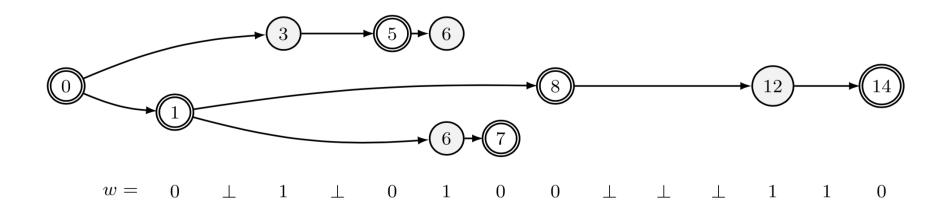
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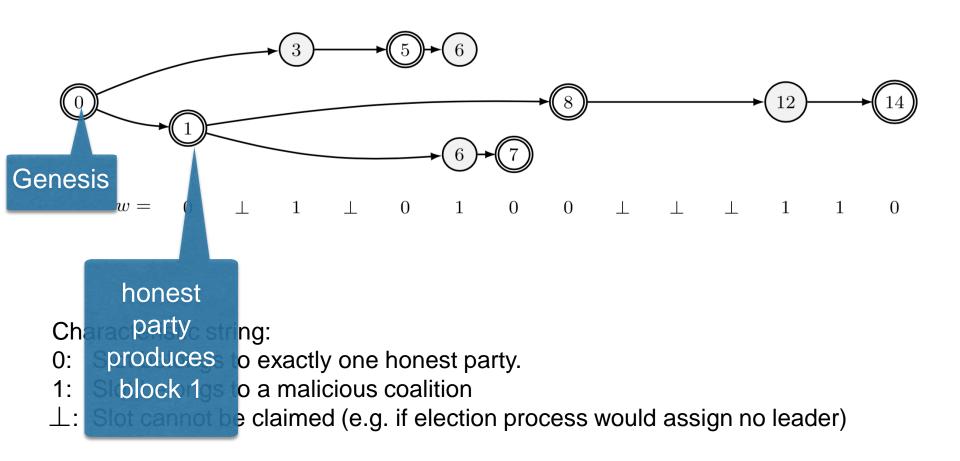


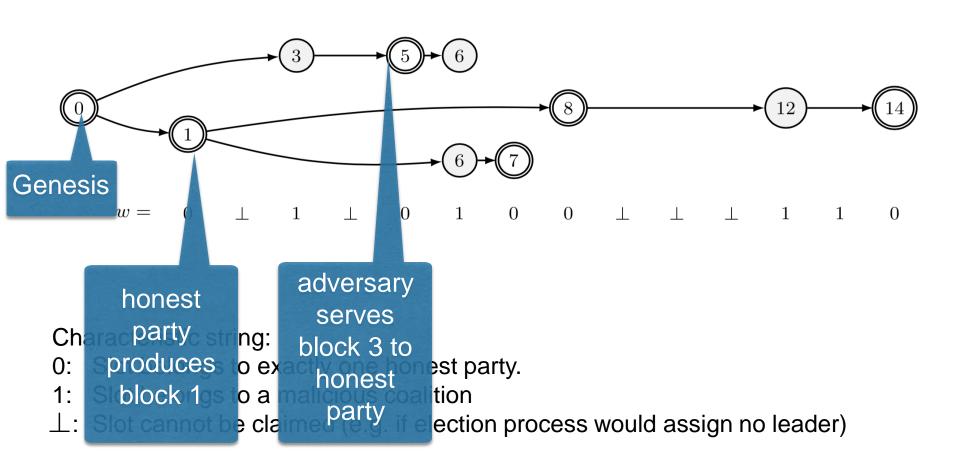
The analysis reveals that the vast majority of strings (under proper conditions) have admissible graphs that translate to well behaved protocol executions.

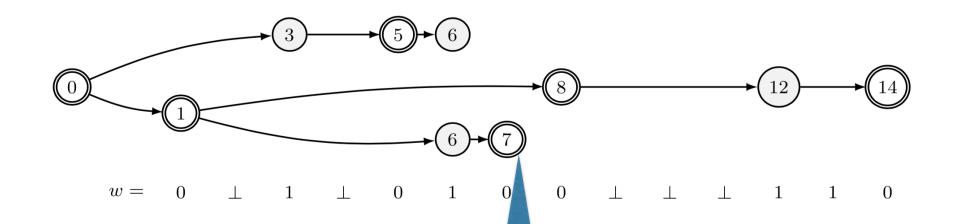


#### Characteristic string:

- 0: Slot belongs to exactly one honest party.
- 1: Slot belongs to a malicious coalition
- ⊥: Slot cannot be claimed (e.g. if election process would assign no leader).







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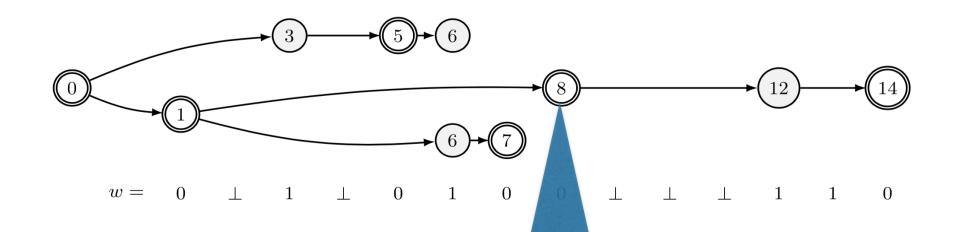
adversary serves

block 6 to

honest

party

d assign no leader)



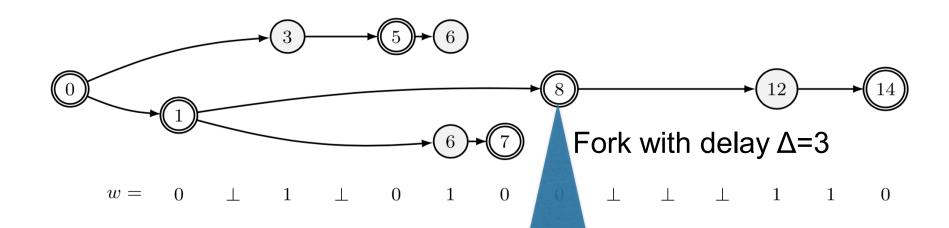
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adversary serves block 1 to honest party (Δ=3)



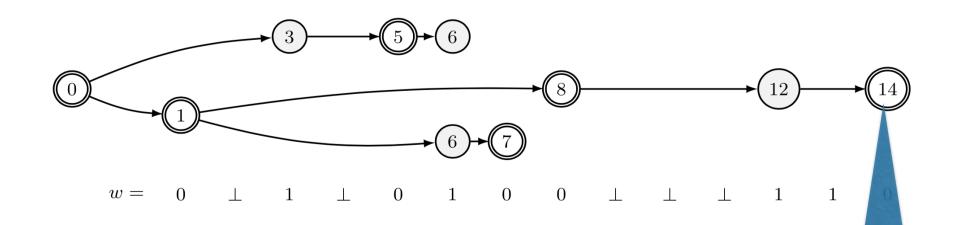
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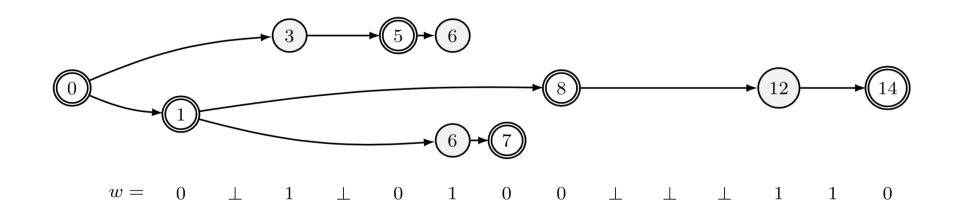
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adversary serves block 12 to honest party

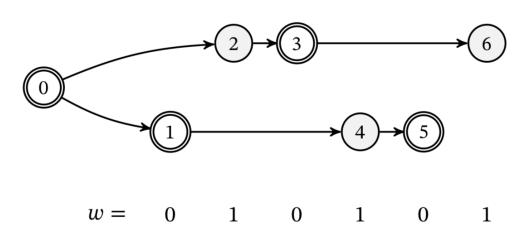


### **Important Property:**

Depth of **honest nodes increases** (from left to right) if more than  $\Delta$  slots apart. (Lower bound on the depth of the fork)

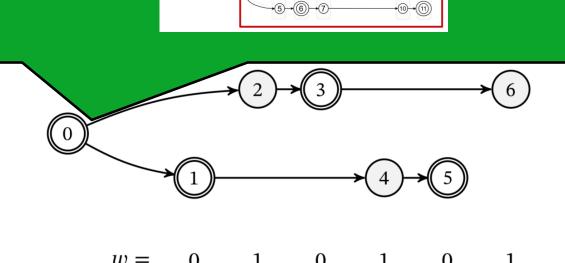
## **Combinatorics of Characteristic Strings**

- Given a characteristic string can we classify the family of forks that it permits?
  - Characteristic string is drawn according to a specific probability distribution: bias toward 0 (by honest-majority assumption).
- Forkable string: those strings that allow a fork with two tines of length equal to the height of the fork.



### Focus on this particular structure:

- → Analysis shows that this is a very unlikely structure to occur (as a function of the length of the sampled string).
- → Note: Also unlikely as a subgraph of any execution, i.e., no execution has such a bad divergence point (and thus we have CP).



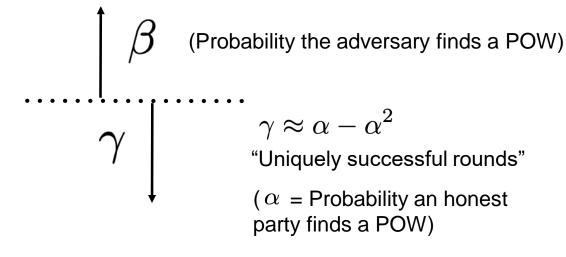
## **Drawing from Bitcoin analysis**

### **Difference:**

- 1.) #PoWs of adversary in time segment
- 2.) **#PoWs of honest parties**in time segment

- 5 4
- 3
- 2
- 1
- 0
- -1
- -2
- -3
- -4
- -5

At the core of the analysis lies a 1D Random Walk



## **Drawing from Bitcoin analysis**

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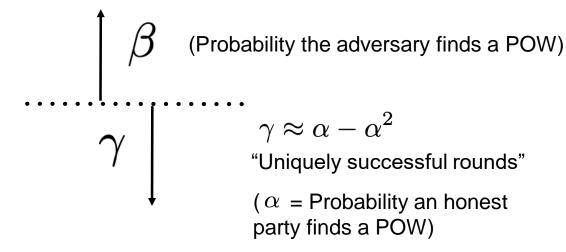
-2

-3

-4

-5

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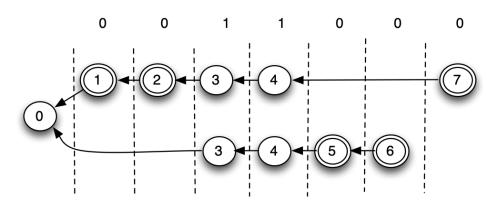
- A favorable step is downwards.
- Such a step is more likely by assumption  $\gamma>eta$  .

#### from PoW to PoS

 Winning a slot for the honest parties (even uniquely) does not necessarily constitute a favorable step in the random walk.

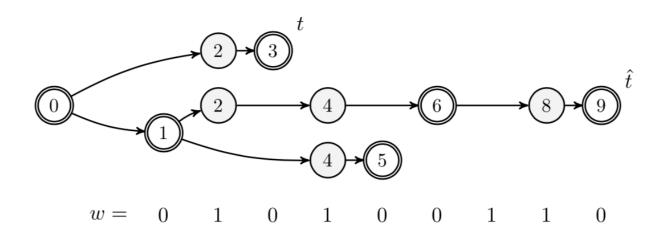
#### "Nothing-at-stake":

The adversary may reuse an opportunity to issue a block in multiple paths of a fork



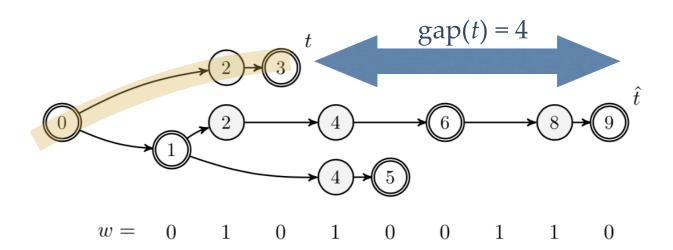
(for simplicity we do only the {0,1} case)

- gap(t): length difference with leading honest node.
- reserve(t): number of adversarial slots after end of t.
- reach(t) := reserve(t) gap(t).



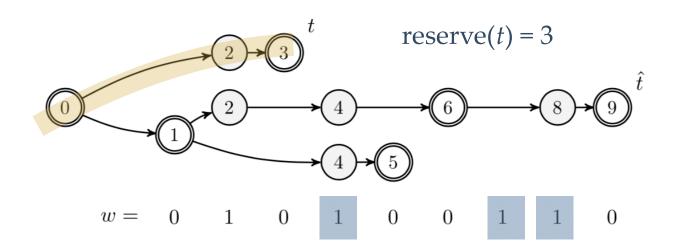


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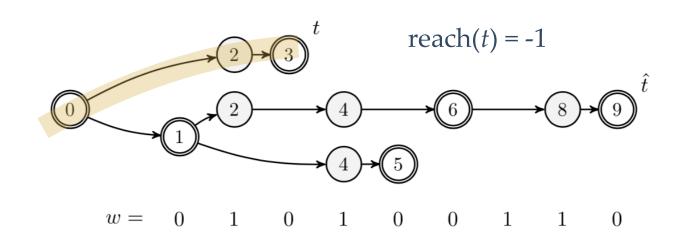




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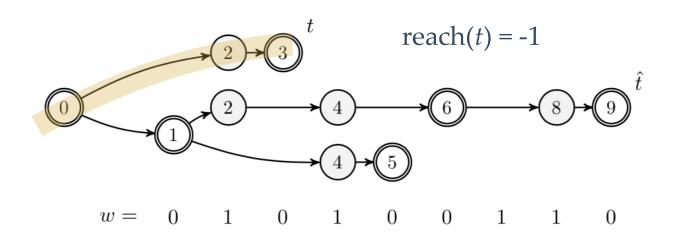
Can the adversary catch up with "longest chain" with this tine?

of interest to

honest node.

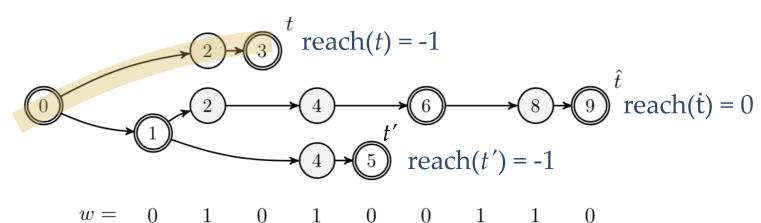
res
 number of adversarial slots after end of t.

reach(t) := reserve(t) - gap(t).



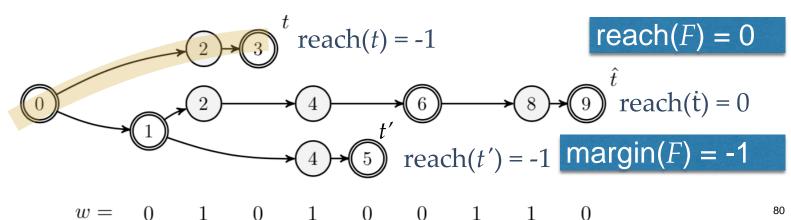
Looking at a fork F in general, we are interested in:

- reach(F): max reach(t)
- margin(F): second highest & disjoint reach(t')



Looking at a **fork F** in general, we are interested in:

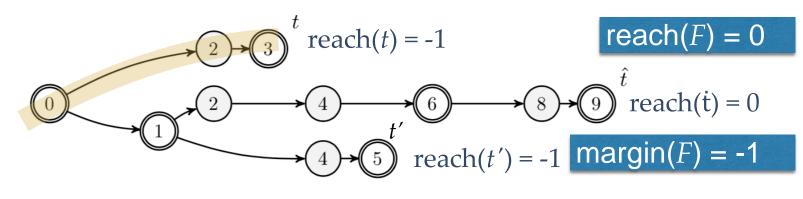
- reach(F): max reach(t)
- margin(F): second highest & disjoint reach(t')



Looki

If second highest allows to catch up with longest: margin non-negative.

- reach(F): Ma.
- margin(F): second highest & disjoint reach(t')



0

## Reach & Margin

Define:

$$\rho(w) = \max_{F} \operatorname{reach}(F)$$
 $\mu(w) = \max_{F} \operatorname{margin}(F)$ 

- Fact: w is forkable (adversary wins) iff  $\mu(w) \ge 0$ .
- We want to prove that the density of forkable strings among all strings is tiny (assuming Hamming weight is below 1/2).
- We consider a 2D random walk defined by the pair  $(\rho(w), \mu(w))$  where w is a binomial random variable.

$$[\rho(w1), \mu(w1)] = [\rho(w) + 1, \mu(w) + 1]$$

$$[\rho(w0), \mu(w0)] = \begin{cases} [\rho(w) - 1, 0] & \rho(w) > \mu(w) = 0 \\ [0, \mu(w) - 1] & \rho(w) = 0 \\ [\rho(w) - 1, \mu(w) - 1] & \text{otherwise} \end{cases}$$

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reach and margin decrement

$$[\rho(w1), \mu(w1)] = [\rho(w) + 1, \mu(w) + 1]$$

$$[\rho(w0),\mu(w0)] = \begin{cases} [\rho(w)-1,0] & \rho(w)>\mu(w)=0\\ [0,\mu(w)-1] & \rho(w)=0\\ [\rho(w)-1,\mu(w)-1] & \text{otherwise} \end{cases}$$
 reach never drops below 0

$$[\rho(w1), \mu(w1)] = [\rho(w) + 1, \mu(w) + 1]$$

margin, by

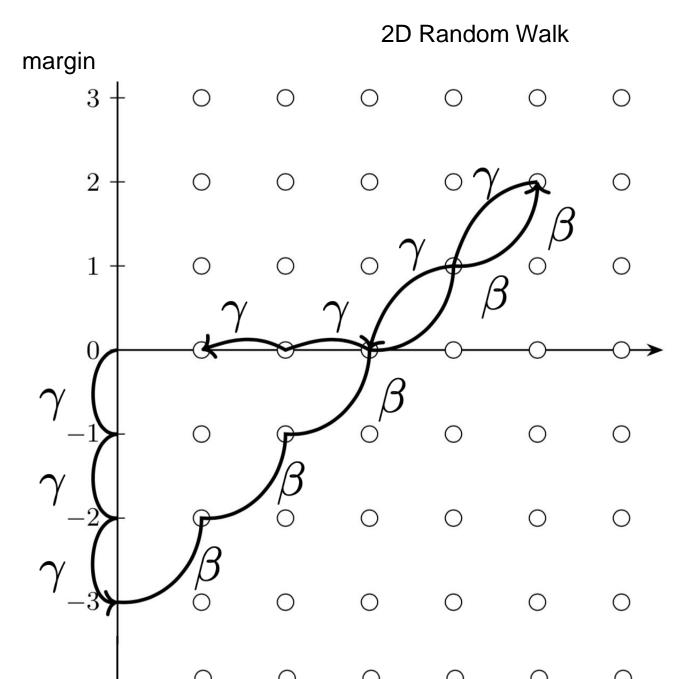
sacrificing reach

$$[\rho(w0),\mu(w0)] = \begin{cases} [\rho(w)-1,0] & \rho(w)>\mu(w)=0 \\ [0,\mu(w)-1] & \rho(w)=0 \\ [\rho(w)-1,\mu(w)-1] & \text{otherwise} \end{cases}$$
 it is possible for the adversary to compensate for the

drops below 0

margin

decrement

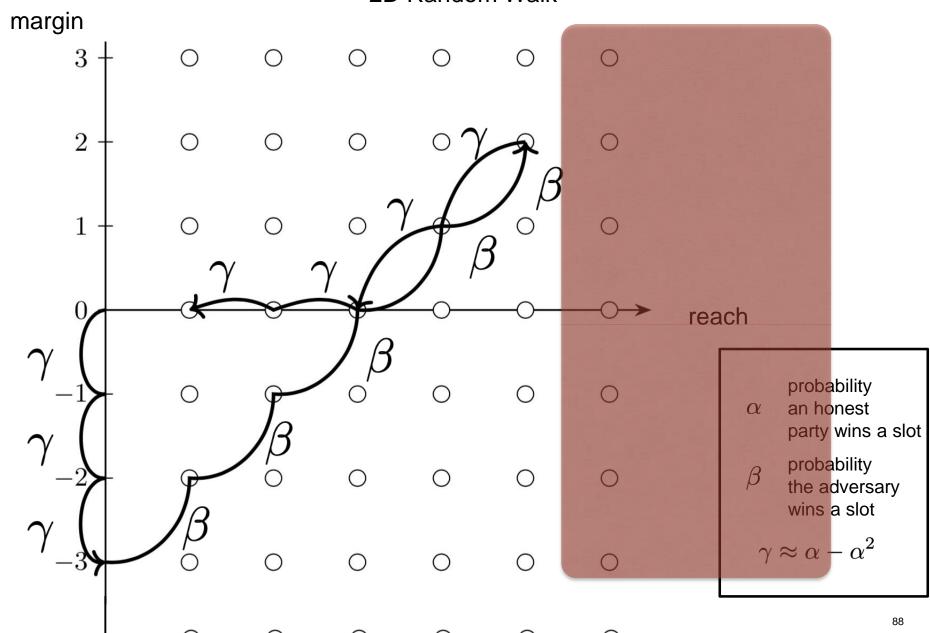


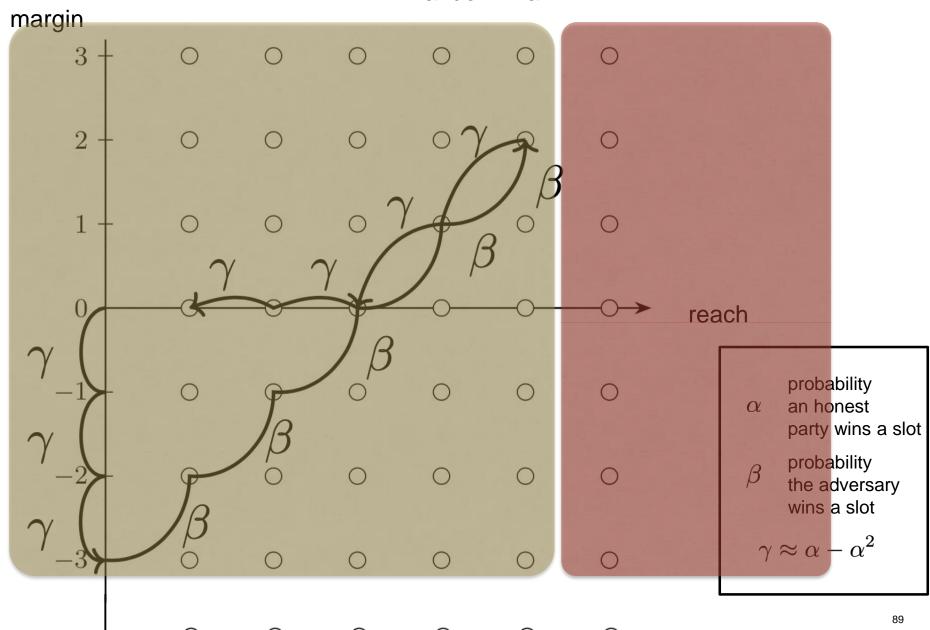
reach

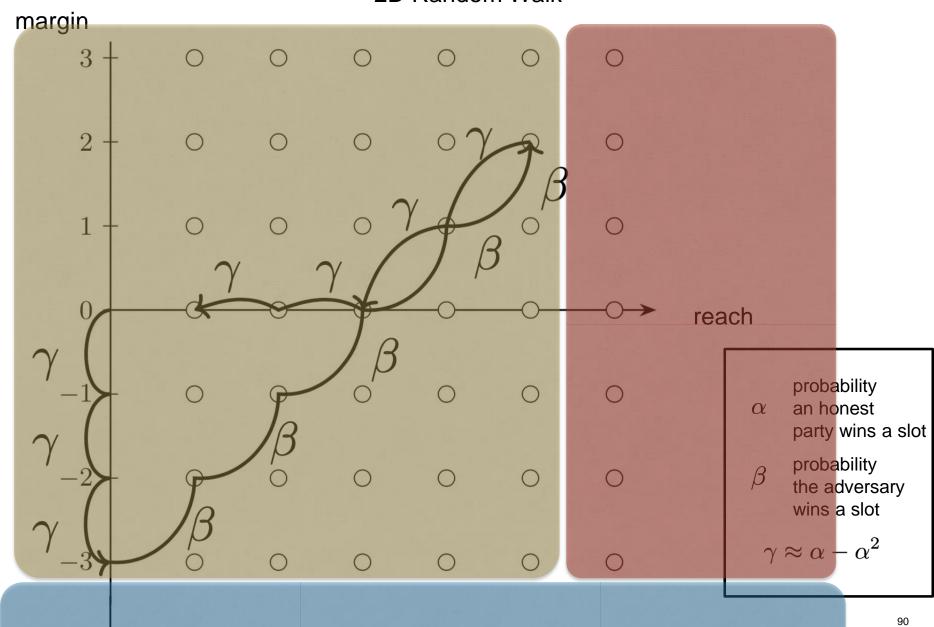
 $\begin{array}{c} \text{probability} \\ \alpha \quad \text{an honest} \\ \text{party wins a slot} \end{array}$ 

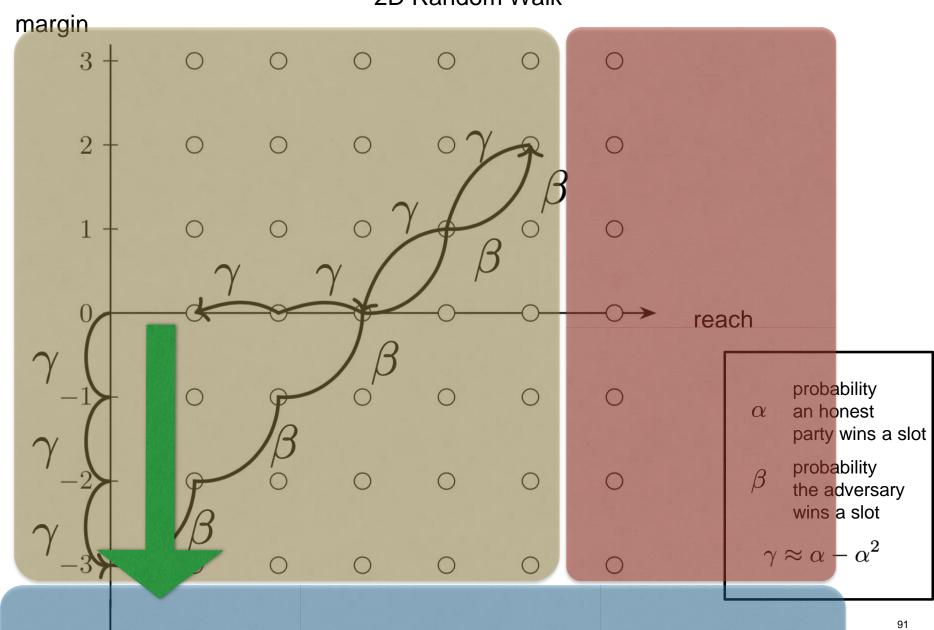
probability the adversary wins a slot

$$\gamma \approx \alpha - \alpha^2$$









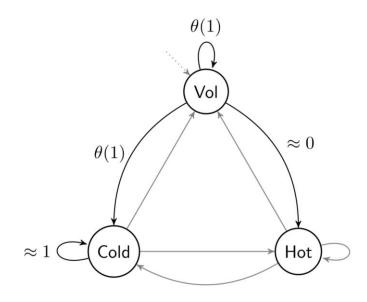
$$R_t = \rho(w_1 \dots w_t)$$
 and  $M_t = \mu(w_1 \dots w_t)$ .

**Hot** We let  $\operatorname{Hot}_t$  denote the event that  $R_{(t)} \geq \delta \sqrt{n}$  and  $M_{(t)} \geq -\delta \sqrt{n}$ .

Volatile We let Vol<sub>t</sub> denote the event that  $-\delta\sqrt{n} \leq M_{(t)} \leq R_{(t)} < \delta\sqrt{n}$ .

Cold We let Cold<sub>t</sub> denote the event that  $M_{(t)} < -\delta\sqrt{n}$ .

$$\begin{split} & \Pr[\mathsf{Cold}_{t+1} \mid \mathsf{Cold}_t] \geq 1 - 2^{-\Omega(\sqrt{n})} \,, \\ & \Pr[\mathsf{Cold}_{t+1} \mid \mathsf{Vol}_t] \geq \Omega(\epsilon) \,, \\ & \Pr[\mathsf{Hot}_{t+1} \mid \mathsf{Vol}_t] \leq 2^{-\Omega(\sqrt{n})} \,. \end{split}$$



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Cold We let Cold

$$R_{(t)}, M_{(t)}$$
:

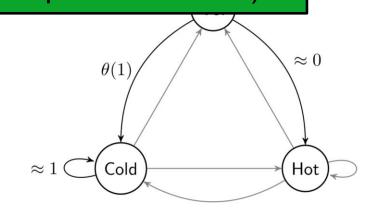
Reach resp. margin after  $t\sqrt{n}$  steps of the random walk ("coarse grained steps of the walk").

**Analy** 

$$\Pr[\mathsf{Cold}_{t+1} \mid \mathsf{Cold}_t] \ge 1 - 2^{-\Omega(\sqrt{n})},$$

$$\Pr[\mathsf{Cold}_{t+1} \mid \mathsf{Vol}_t] \ge \Omega(\epsilon)$$
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$$\Pr[\mathsf{Hot}_{t+1} \mid \mathsf{Vol}_t] \leq 2^{-\Omega(\sqrt{n})}$$
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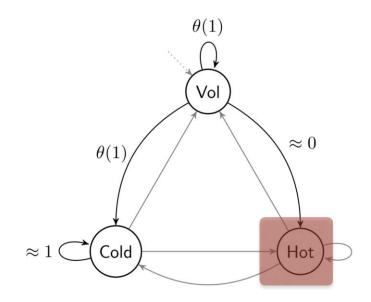
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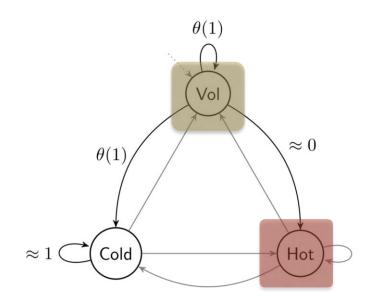
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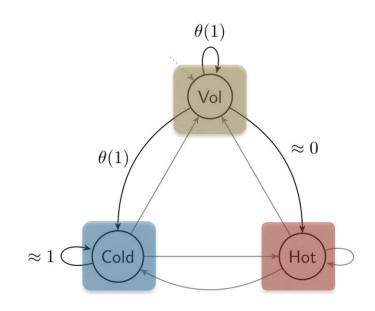
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Vola

Cold

An improved analysis shows an error bound of

$$2^{-\Omega(n)}$$

"The Combinatorics

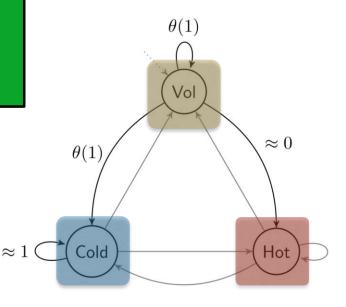
"The Combinatorics of the Longest-Chain Rule: Linear Consistency for Proof-of-Stake Blockchains" by Erica Blum and Aggelos Kiayias and Cristopher Moore and Saad Quader and Alexander Russell.

$$\Pr[\mathsf{Cold}_{t+1} \mid \mathsf{Cold}_t] \ge 1-2$$

$$\Pr[\mathsf{Cold}_{t+1} \mid \mathsf{Vol}_t] \ge \Omega(\epsilon)$$
,

$$\Pr[\mathsf{Hot}_{t+1} \mid \mathsf{Vol}_t] \leq 2^{-\Omega(\sqrt{n})}$$
.

$$\sqrt{n} \le M_{(t)} \le R_{(t)} < \delta \sqrt{n}.$$
  
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Hot

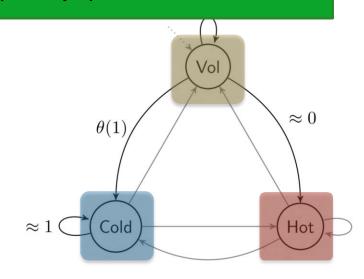
Vola

Cold

#### **Conclusion:**

- Characteristic string not forkable (w.h.p.)
  - → No long diverging paths
- Common prefix achieved (w.h.p.)

$$\begin{split} & \Pr[\mathsf{Cold}_{t+1} \mid \mathsf{Cold}_t] \geq 1 - 2^{-\Omega(\sqrt{n})} \,, \\ & \Pr[\mathsf{Cold}_{t+1} \mid \mathsf{Vol}_t] \geq \Omega(\epsilon) \,, \\ & \Pr[\mathsf{Hot}_{t+1} \mid \mathsf{Vol}_t] \leq 2^{-\Omega(\sqrt{n})} \,. \end{split}$$

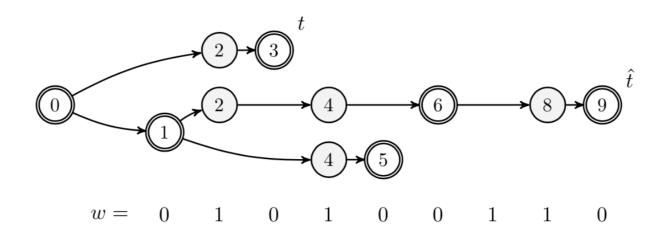


## **Also: Chain Growth & Quality**

Chain Quality: any sufficiently long section along a (viable) tine must contain an honest node with overwhelming probability.

 $(\rightarrow$  Otherwise, #0's < #1's)

Chain Growth: The #0's support growth and by the above, the growth is reflected in any viable tine (with a small discount).



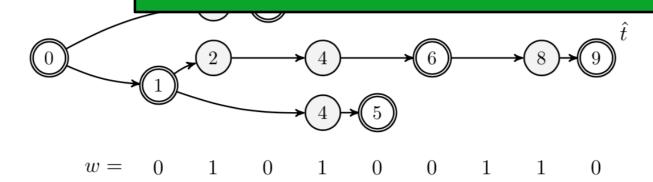
## **Also: Chain Growth & Quality**

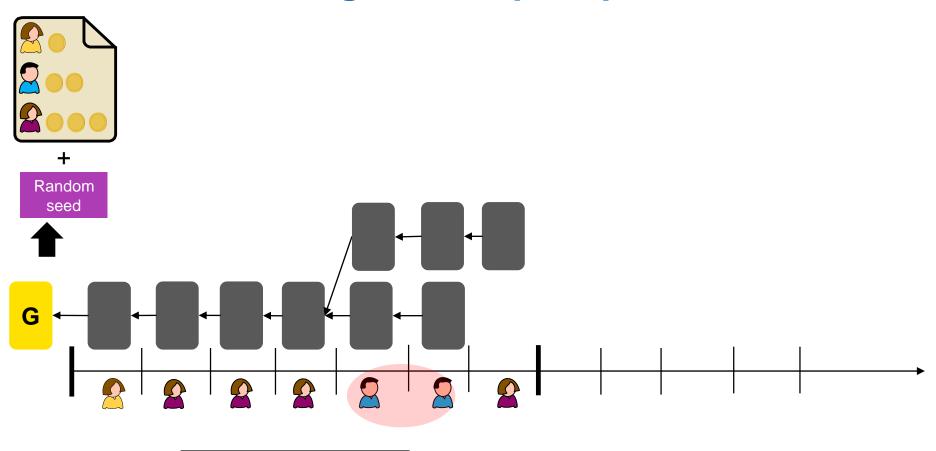
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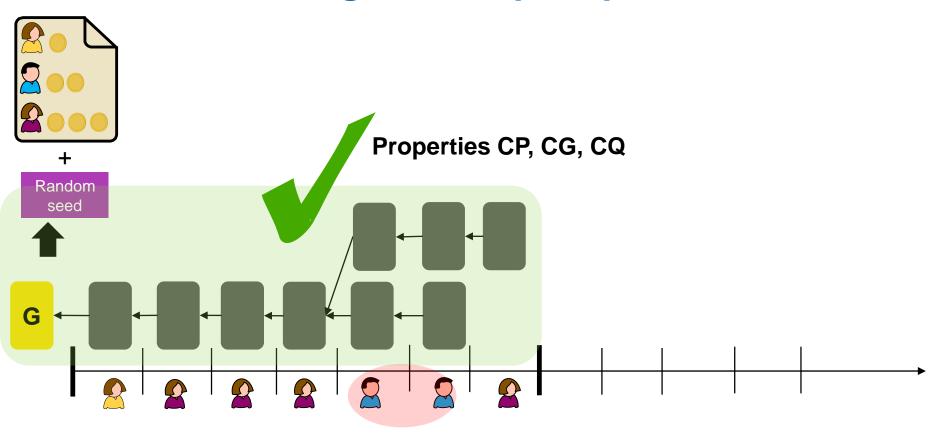
Chain Growth: growth is reflect

Viable tines: Correspond to chains that are long enough to be adopted by an honest party at a given time.

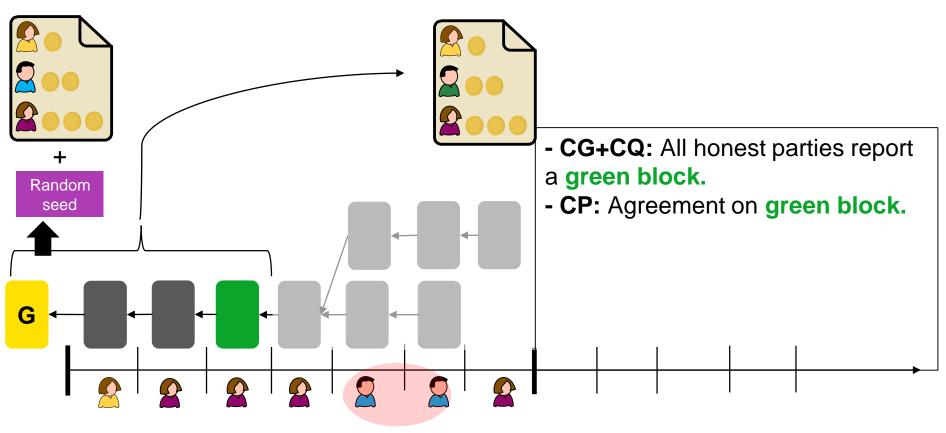




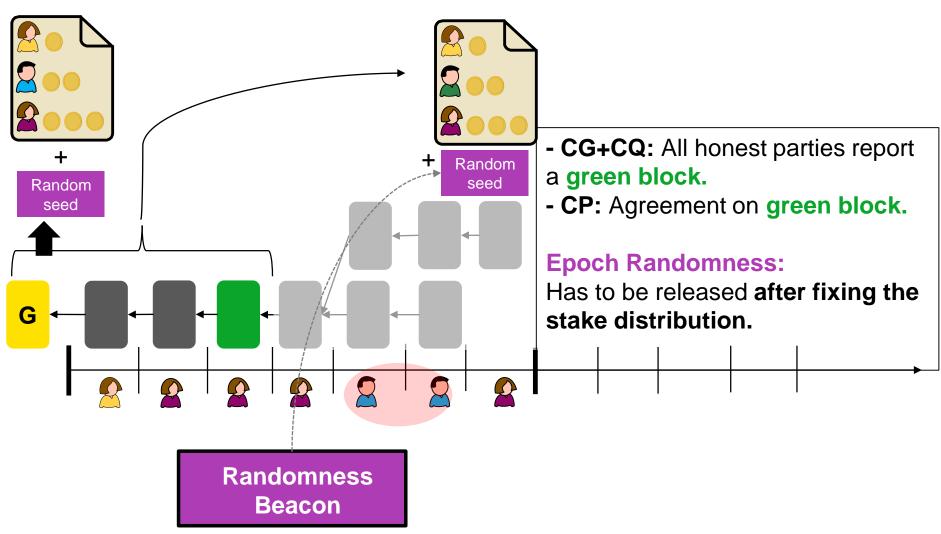
Randomness Beacon



Randomness Beacon

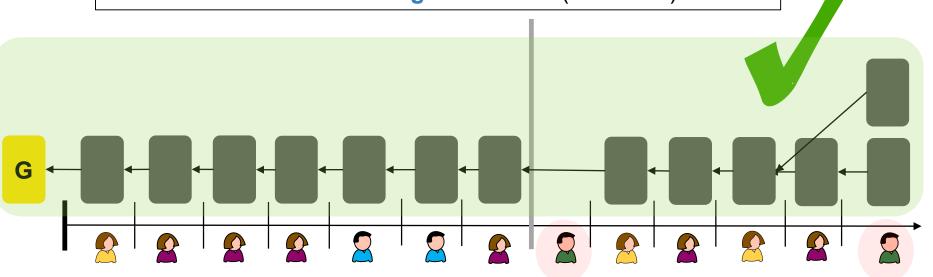


Randomness Beacon



#### "Smooth Epoch Boundaries"

Distribution of entire characteristic string is uniquely defined for this execution and dominated by a binomial distribution favoring 0's over 1's (as before).



#### **Incentives**

#### How to make honest parties participate?

- Costs
  - Such as verifying transactions, packaging them in the right order.
- Rewards
  - Such as collecting fees.

**Problem:** Pure chain quality underrepresents the honest parties' effort: Effort in maintaining the inputs is not rewarded proportionally.

#### **Incentives**

#### How to make honest parties participate?

- Costs
  - Such as verifying transactions, packaging them in the right order.
- Rewards
  - Such as

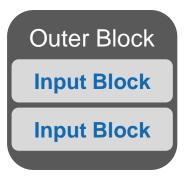
#### **Key Idea:**

Main effort is related to input contribution → Declare it to be a separate task.

### **Incentives**

### **Solution: Input Endorsers**

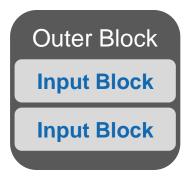
- Each slot elects an additional stakeholder (or a set of stakeholders) to contribute inputs.
  - Using a parallel lottery.
  - Like the 2-for-1 mechanism in PoW as in GKL analysis or Fruitchains.
- Endorsed inputs are permitted in the blockchain any time within a small window following and inclusive the slot that elects them.



### **Incentives**

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- Endorsed inputs are permitted in the blockchain any time within a small window following and inclusive the slot that elects them.



→ Overall #Input blocks proportional to stake.

→ Protocol becomes a Nash equilibrium for an appropriate reward function (that rewards input blocks in an aggregate fashion over a sequence of blocks).

#### **Ouroboros BFT**

Ouroboros "Classic" (Crypto 17)

**Ouroboros BFT** 

Ouroboros
Praos
(Eurocrypt 2018)

An extremely simple BFT protocol that follows from the Ouroboros Classic analysis

Ouroboros Genesis (CCS 2018)



Ouroboros Crypsinous (S&P 2019)

Ouroboros Chronos (In submission, 2019) Ouroboros with Privacy

### **Ouroboros BFT**

Ouroboros "Classic" (Crypto 17)



**Ouroboros BFT** 

Ouroboros

An extremely simple BFT protocol that follows from the Ouroboros Classic analysis

#### **Central Observation:**

A characteristic string (assume binary) with Hammingweight less than 1/3 is not forkable.

Ouroboros with Privacy

Ouroboros
Chronos
(In submission, 2019)

Ouroboros "Classic" (Crypto 17)

Ouroboros Praos (Eurocrypt 2018)

Ouroboros Genesis (CCS 2018)

Ouroboros
Chronos
(In submission, 2019)

Semi-adaptive adversaries, synchrony Strong mathematical framework



+ Network Delay ("semi-synchronous")



- + Full dynamic availability
- + Bootstrapping from Genesis



- + Only based on same-speed assumption.
- + Bootstrapping state and time from genesis
- = PoS blockchain in the DA setting without global clocks.

Ouroboros "Classic" (Crypto 17)



Semi-adaptive adversaries, synchrony Strong mathematical framework

**Ouroboros** 

+ Adaptive Adversaries

+ Network Pri-synchronous")

### Stronger cryptographic primitives needed:

- To enable private lottery
- To fully mitigate adaptive corruptions

Ouroboros
Chronos
(In submission, 2019)



- + Only based on same-speed assumption.
- + Bootstrapping state and time from genesis

= PoS blockchain in the DA setting without global clocks.

Ouroboros "Classic" (Crypto 17)

Ouroboros Praos (Eurocrypt 2018)

Ouroboros Genesis Semi-adaptive adversaries, synchrony Strong mathematical framework

- + Adaptive Adversaries
- + Network Delay ("semi-synchronous")
- + Full dynamic availability
- + Bootstrapping from Genesis

Newcomers should be able to join the system without the extra help of existing parties.

d assumption. ne from genesis

setting

Ouroboros "Classic" (Crypto 17)

Ouroboros
Praos
(Eurocrypt 2018)

Ouroboros Genesis (CCS 2018)

Ouroboros
Chronos
(In submission, 2019)

Semi-adaptive adversaries, synchrony Strong mathematical framework



+ Network Delay ("semi-synchronous")

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+ Bootstrapping from Genesis

**Genesis Security Proof** 



- + Only based on same-speed assumption.
- + Bootstrapping state and time from genesis

= PoS blockchain in the DA setting without global clocks.

# Cryptographic Tools to Protect against Adaptive Adversaries

# Cryptographic Tools to Protect against Adaptive Adversaries

**Verifiable random functions (VRF)** – with unpredictability under malicious key generation.

(VRF.Gen, VRF.Eval, VRF.Verify)

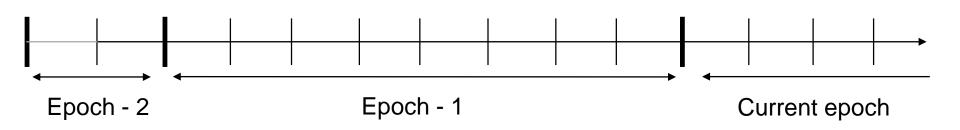
- → Output appears pseudo-random (for a new input)
- → Input and output are verifiably tied together
- → Output cannot be biased by crafting strange keys
- → Purpose: Allow private leader-election and thereby a more realistic attacker model (mitigate adaptive attacks).

# Cryptographic Tools to Protect against Adaptive Adversaries

#### **Key evolving signature scheme (KES)**

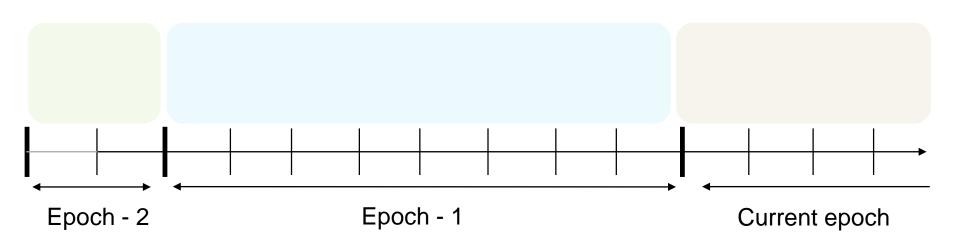
(KES. Gen, KES. Sign, KES. Update, KES. Verify)

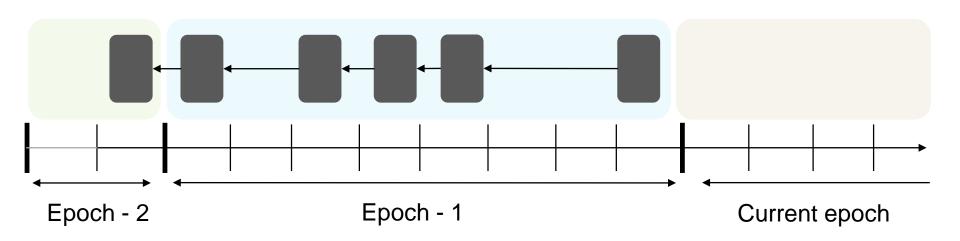
- → Operates as normal signature scheme (unforgeable)
- → Key updates: All values signed "in the past" remain unforgeable even if party gets corrupted after the update.
- → Purpose: Protect previous actions and thereby allow realistic corruption model (tolerate "immediate corruptions").

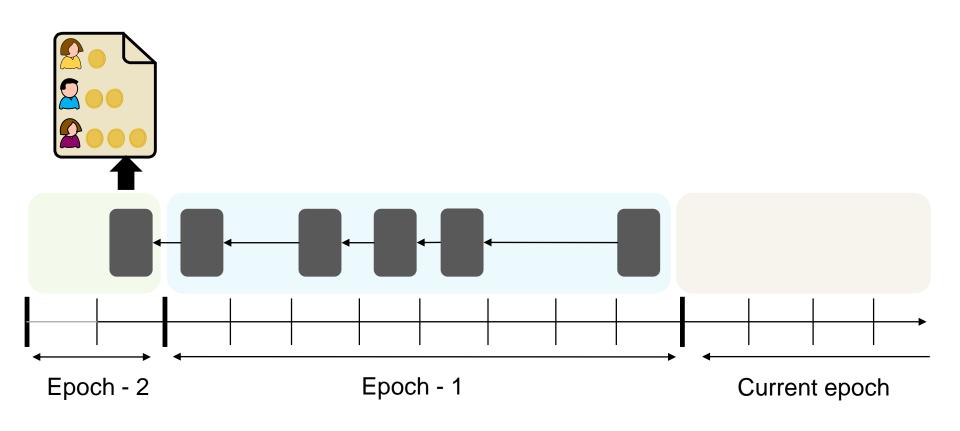


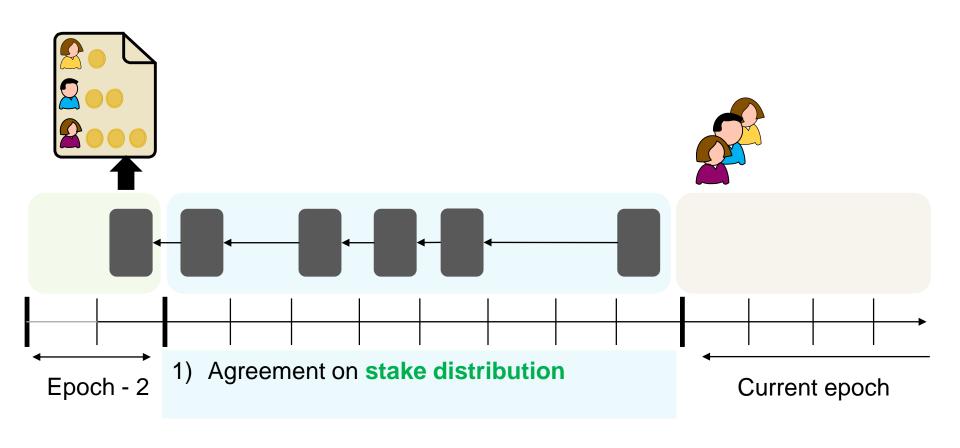
- In each slot, each party evaluates slot-leadership.

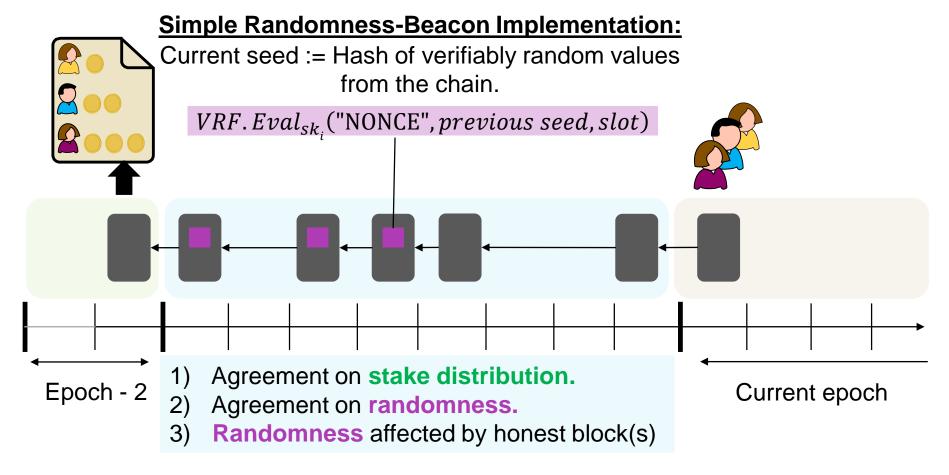
  Private election, proportional to stake, including recent randomness from the chain
- A slot leader extends a chain by creating the block for this slot.

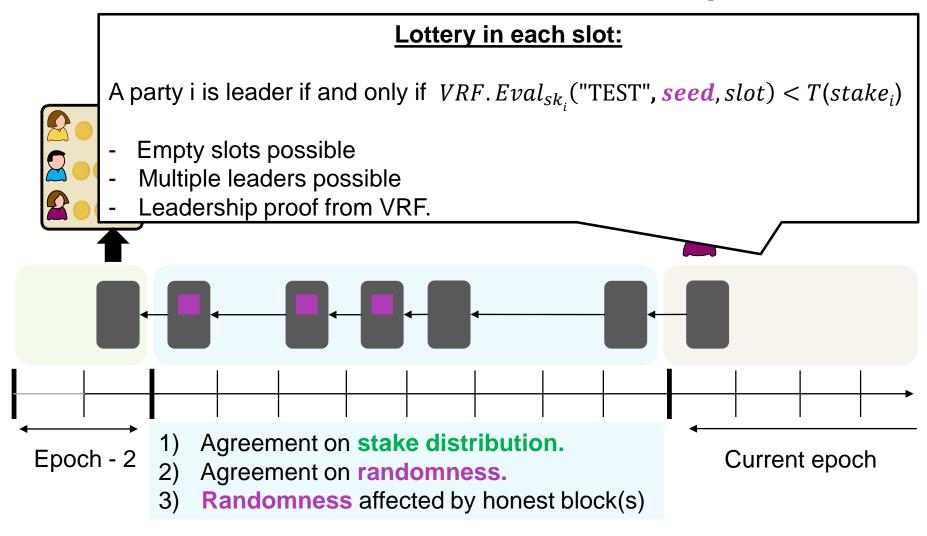










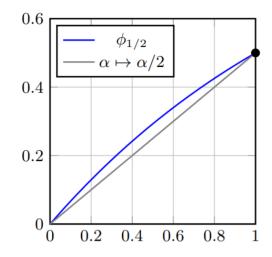


# **Ouroboros Praos/Genesis: Details on Leader Election**

 $VRF.Eval_{sk_i}$ ("TEST", **seed**, slot)  $< T(stake_i)$ 

$$\rightarrow T(stake_i) = 2^{\ell_{VRF}} \varphi_f(rel.stake_i)$$

$$\Rightarrow \qquad \varphi_f(x) = 1 - (1 - f)^x$$

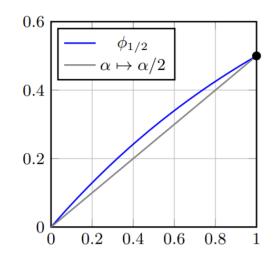


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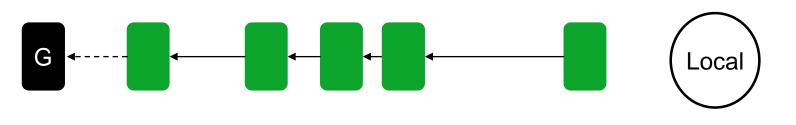
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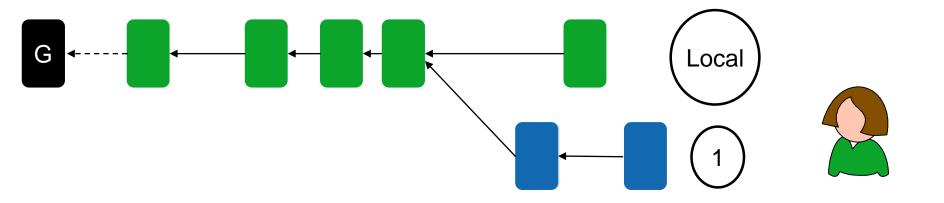
Some remarks:

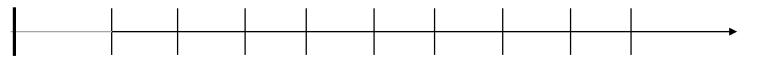
- 1.) Active slot coefficient:  $\varphi_f(1) = f$ ; slot empty with prob. 1 f.
- 2.) Independent aggregation property:  $1 \varphi_f\left(\sum_i x_i\right) = \prod_i (1 \varphi_f(x_i))$ 
  - → Probability of leadership independent of distribution to addresses.
  - $\rightarrow$  The concave (and subadditive as  $\varphi_f(0) = 0$  ) property eases the analysis.

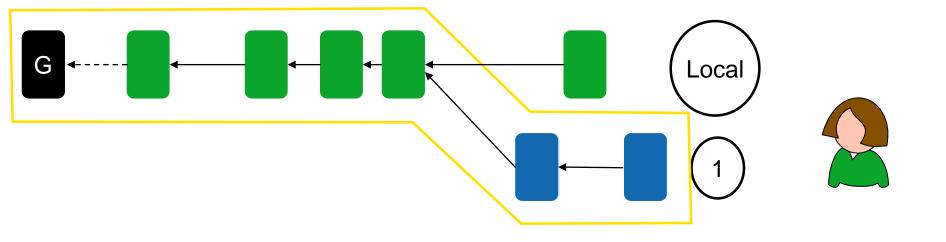


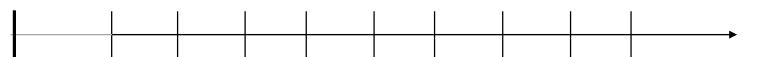


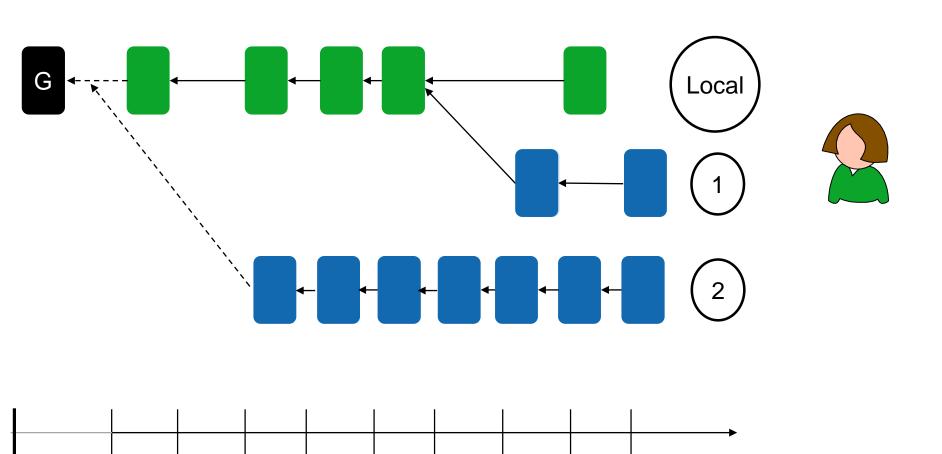




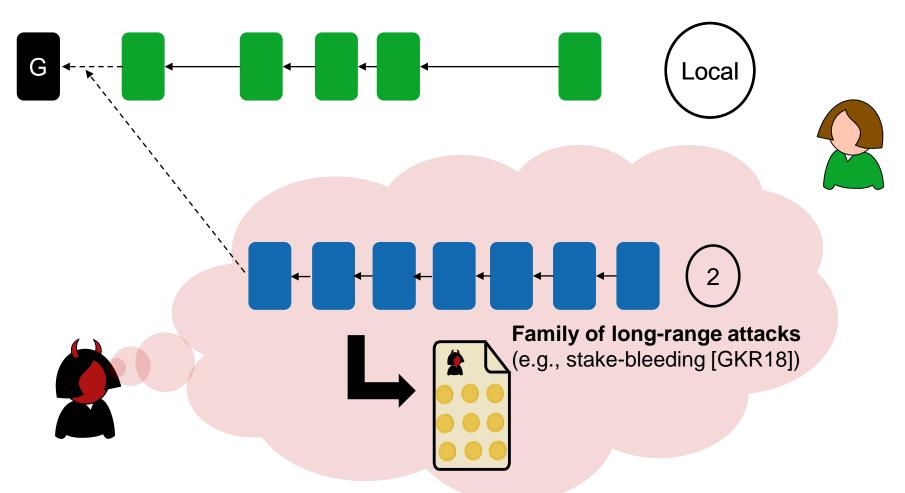








# **Attention: Longest Chain Rule Does not Work**



Chain Selection Rule [e.g., DGKR18]:

Adopt a valid new chain...

 if it is longer and does not fork by more than k blocks from local chain.

Otherwise, keep local chain.









# At first sight...

- ... it seems one would require one of the following:
- 1.) Online parties maintain a moving checkpoint
   → Joining parties need advice.
- 2.) A fixed and known lower bound on participation
  - → No flexible participation, protocol might be stalled.

Ouroboros
"Classic"
(Crypto 17)

Ouroboros Praos (Eurocrypt 2018)

Ouroboros Genesis (CCS 2018)

Ouroboros
Chronos
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- + Network Delay ("semi-synchronous")



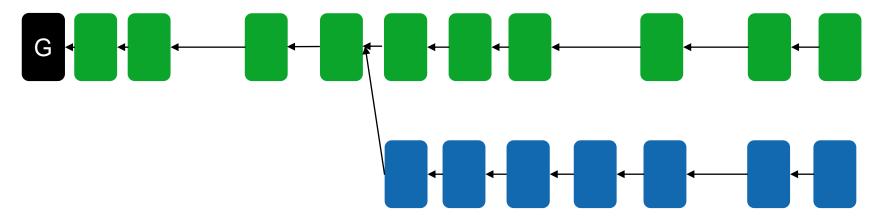
- + Full dynamic availability
- + Bootstrapping from Genesis



- + Only based on same-speed assumption.
- + Bootstrapping state and time from genesis
- = PoS blockchain in the DA setting without global clocks.

- ... it seems one would require one of the following:
- 1.) Online parties maintain a moving checkpoint
   → Joining parties need advice
- 2.) A fixed and known lower bound on participation
  - No flexible participation, protocol might be stalled.
    We do not require either of these!

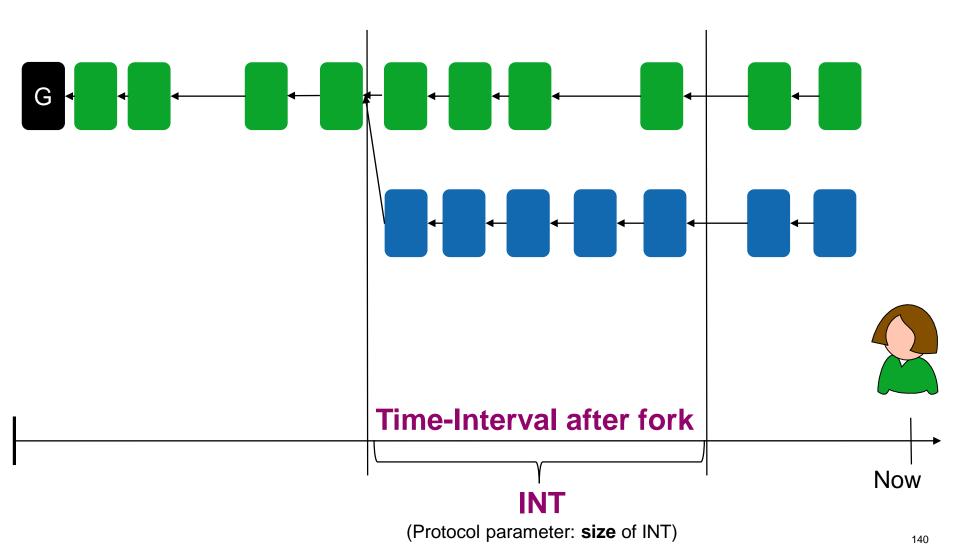
Thanks to a more involved Chain-Selection Rule





139

Now



#### **Genesis Rule:**

Adopt a valid new chain...

- if it is longer and does not fork by more than k blocks from local chain.
- or if it forks by more than k blocks but has higher block density on interval INT.

Otherwise, keep local chain.



Now

Time interval after fork

INT

Ouroboros "Classic" (Crypto 17)

Ouroboros
Praos
(Eurocrypt 2018)

Ouroboros Genesis (CCS 2018)

Ouroboros
Chronos
(In submission, 2019)

Semi-adaptive adversaries, synchrony Strong mathematical framework



+ Network Delay ("semi-synchronous")

+ Full dynamic availability

+ Bootstrapping from Genesis

**Genesis Security Proof** 

- + Only based on same-speed assumption.
- + Bootstrapping state and time from genesis

= PoS blockchain in the DA setting without global clocks.





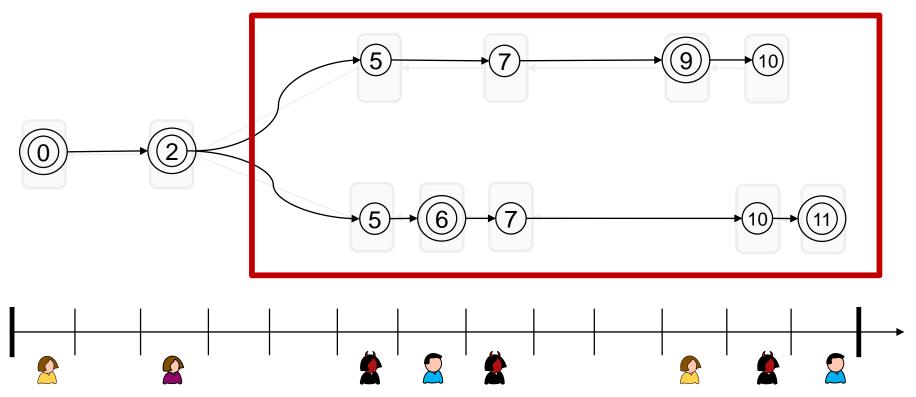
# Roadmap of Security Proof of Genesis

- Security of Ouroboros Genesis with the old chain selection rule (=Praos) and dynamic participation but no newly joining parties.
- Security for joining parties: new Genesis rule in action.

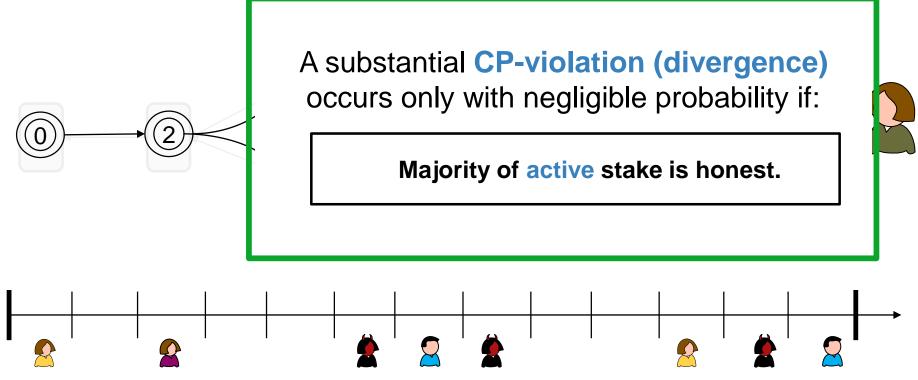
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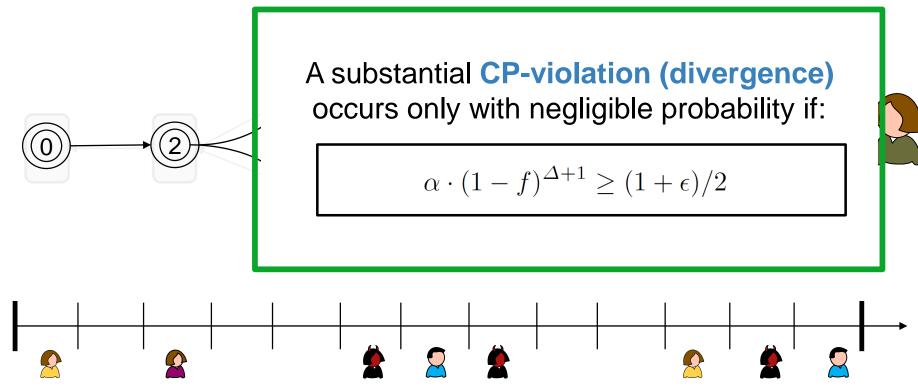
Recall the Fork abstraction:



Recall the Fork abstraction:



Recall the Fork abstraction:



- Dynamic Participation: Dependent variables, biased lottery in favor of honest parties → Martingales
- We show Common Prefix, Chain Growth, Chain Quality
- Realizes the ledger (composable analysis)

## Roadmap of Security Proof of Genesis

- Security of Ouroboros Genesis with the old chain selection rule (=Praos) and dynamic participation but no newly joining parties.
- Security for joining parties: new Genesis rule in action.

## **Security of the Genesis Rule**

#### Claim 1:

If a party is always up-to-date and using the Genesis chainselection rule, she will never adopt a chain that forks by more than k blocks (compared to her local chain in any round).

#### Claim 2:

Using the Genesis chain-selection rule, a newly joining party will adopt a recent chain with large common prefix w.r.t. honest parties. No other advice than the genesis block is needed.

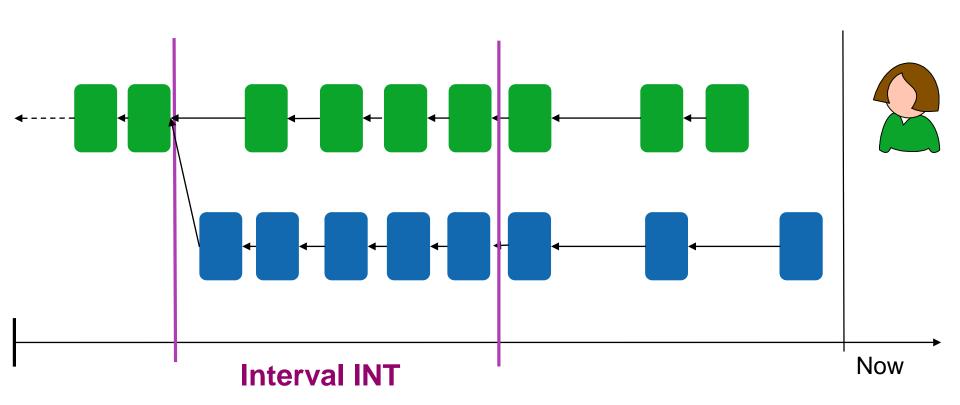
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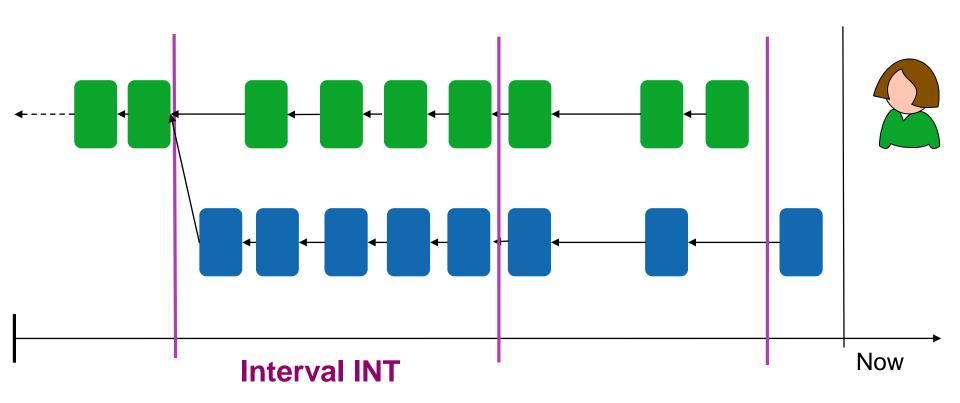
#### Claim 1:

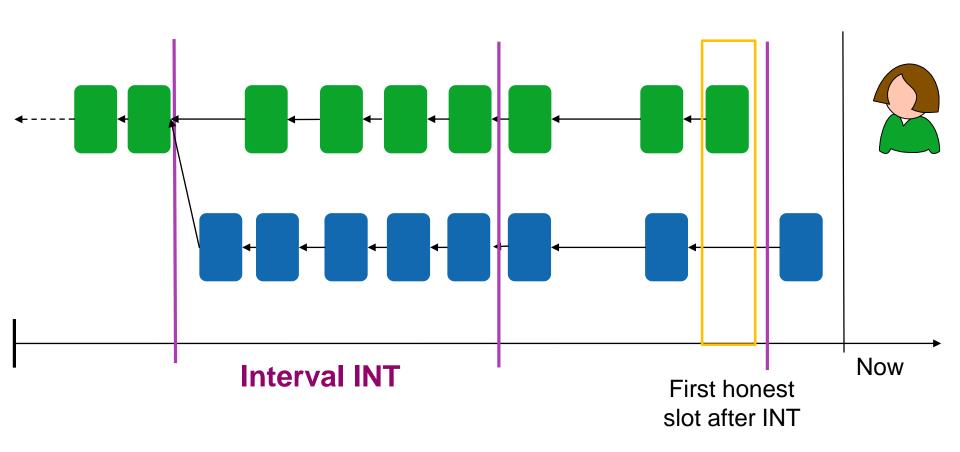
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#### Claim 2:

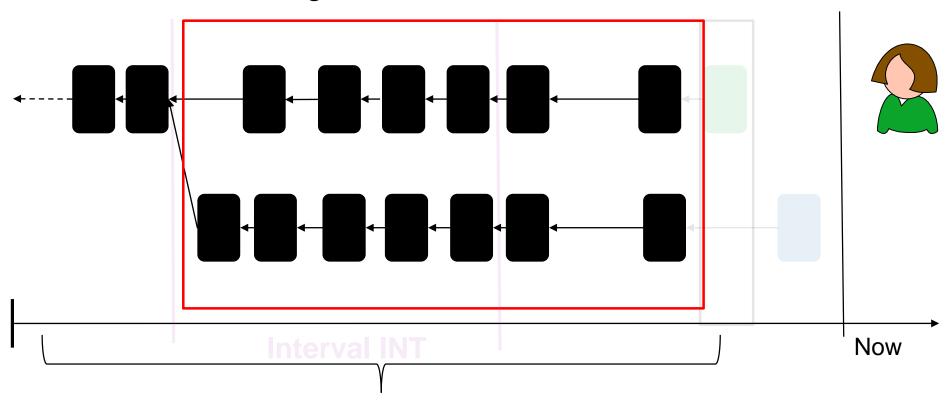
Using the Genesis chain-selection rule, a newly joining party will adopt a recent chain with large common prefix w.r.t. honest parties. No other advice than the genesis block is needed.







A substantial divergence! Hence, situation does not occur.



Covered by previous "Praos" analysis.

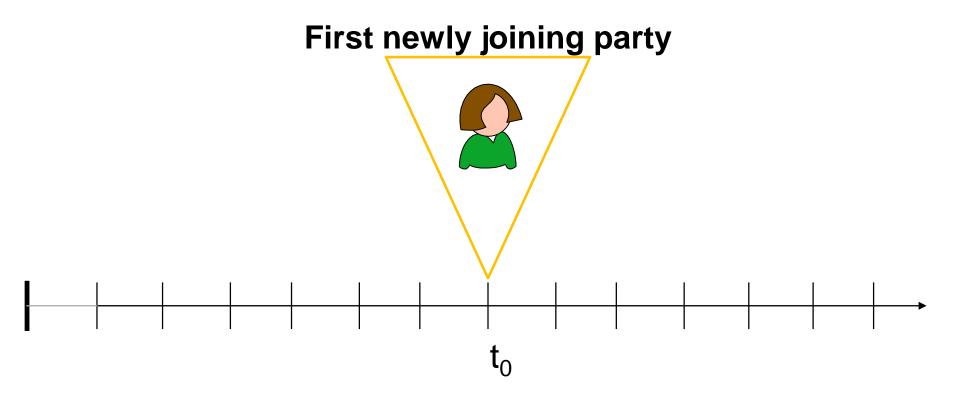
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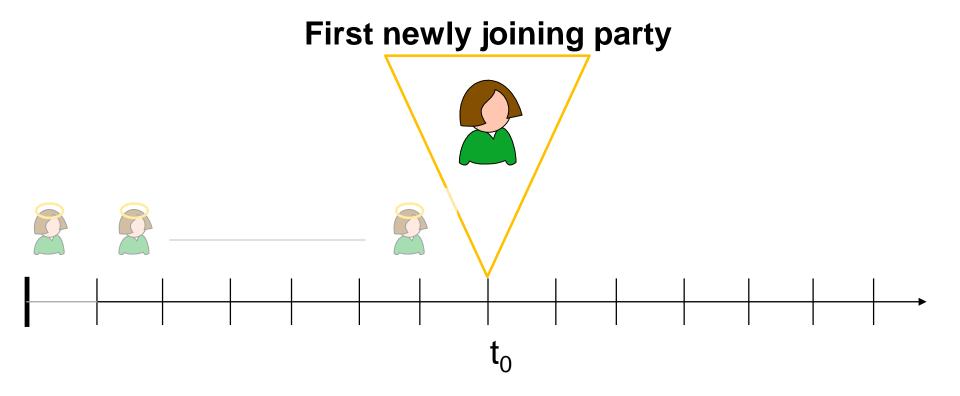
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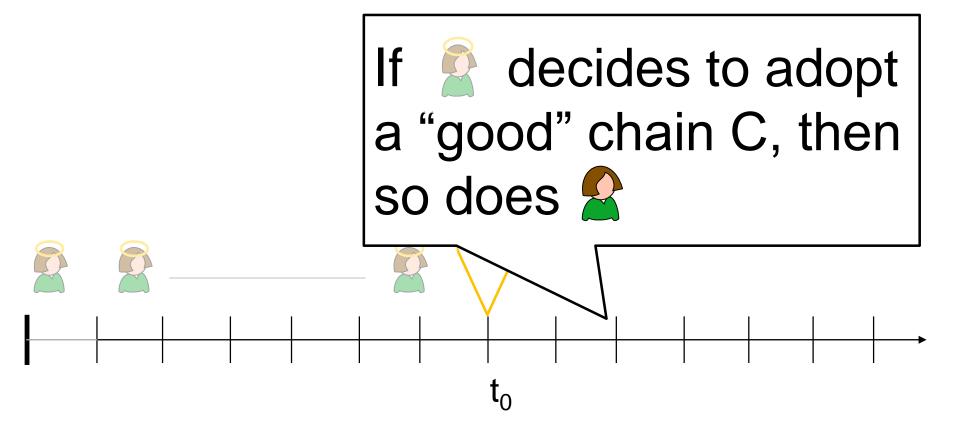
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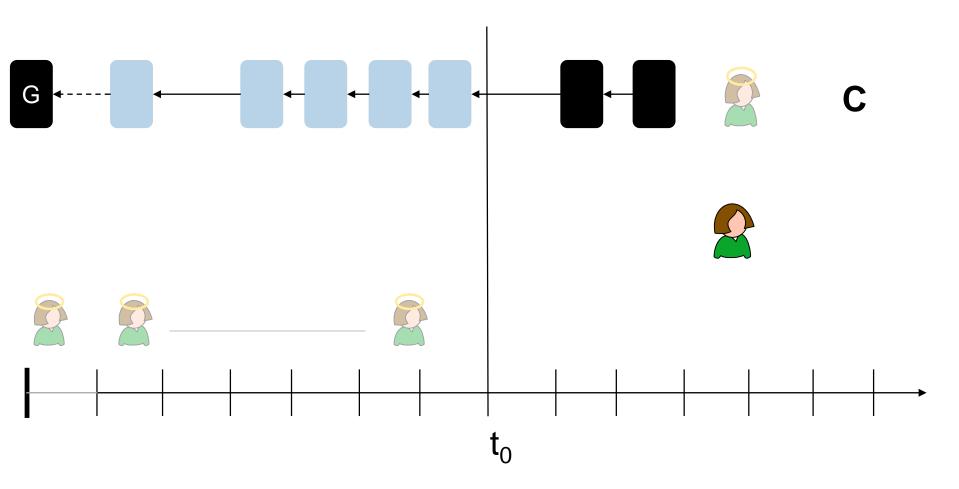
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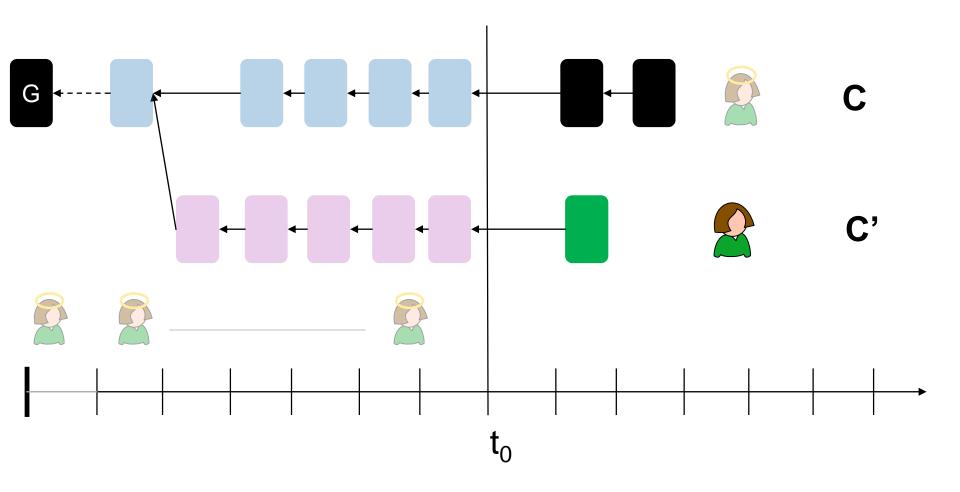
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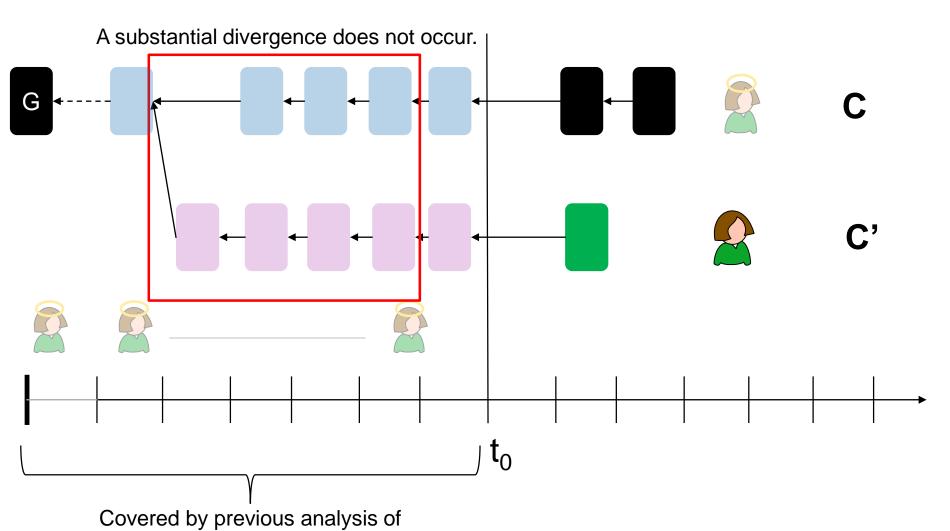












new chain-selection rule

# **Privacy in Ouroboros: Crypsinous**

Ouroboros "Classic" (Crypto 17)



**Ouroboros BFT** 

Ouroboros
Praos
(Eurocrypt 2018)

Ouroboros Genesis (CCS 2018)



Ouroboros Crypsinous (S&P 2019)

Ouroboros Chronos

(In submission, 2019)

Genesis with Privacy

# **Privacy in Ouroboros: Crypsinous**

#### **Problem Summary:**

Public verifiability of leader schedule vs.

Hide amount of stake possessed

(Eurocrypt 2016)

Ouroboros Genesis (CCS 2018)

Ouroboros
Chronos
(In submission, 2019)

Ouroboros Crypsinous (S&P 2019)

Genesis with Privacy

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Ouroboros Chronos

(In submission, 2019)

Genesis with Privacy

- Zero-knowledge proof systems
- SNARKs

## **Ouroboros: Real-World Implementations**

Ouroboros "Classic" (Crypto 17)

**Ouroboros BFT** 

Ouroboros
Praos
(Eurocrypt 2018)

Ouroboros Genesis (CCS 2018)

Cardano is running on Ouroboros PoS and other companies are implementing versions of it.

Ouroboros
Chronos
(In submission, 2019)

#### **Ouroboros – Chronos**

Ouroboros "Classic" (Crypto 17)

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Ouroboros Chronos (In submission, 2019) Semi-adaptive adversaries, synchrony Strong mathematical framework

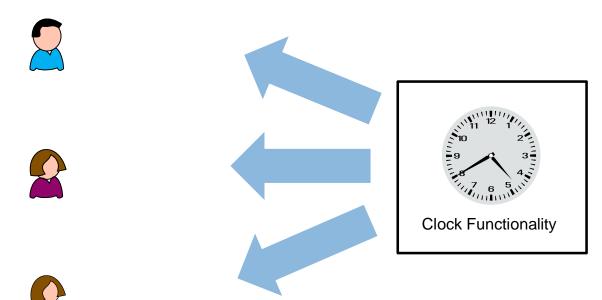
- + Adaptive Adversaries
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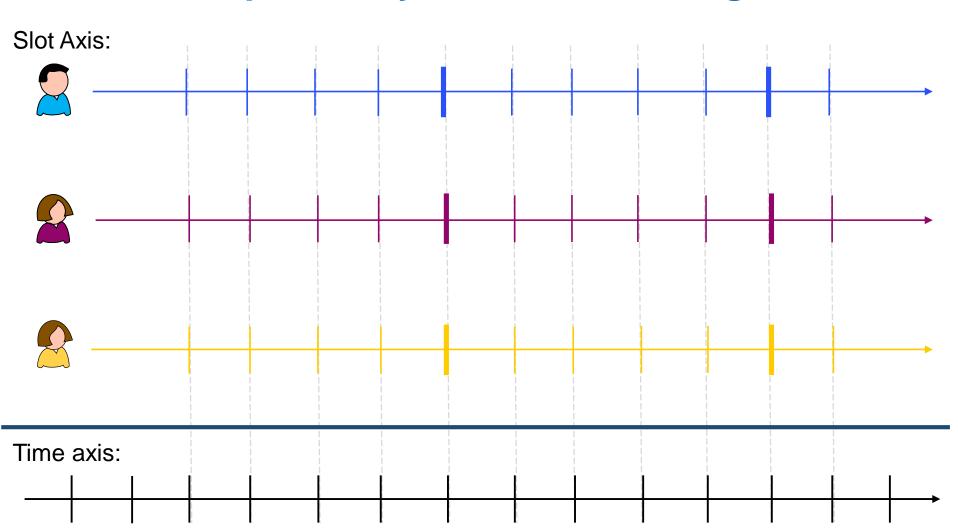
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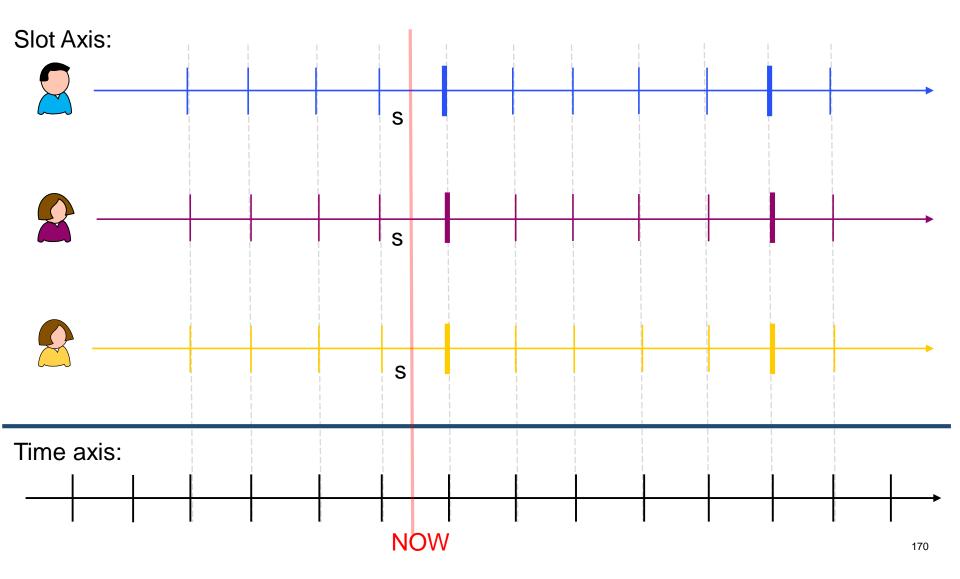
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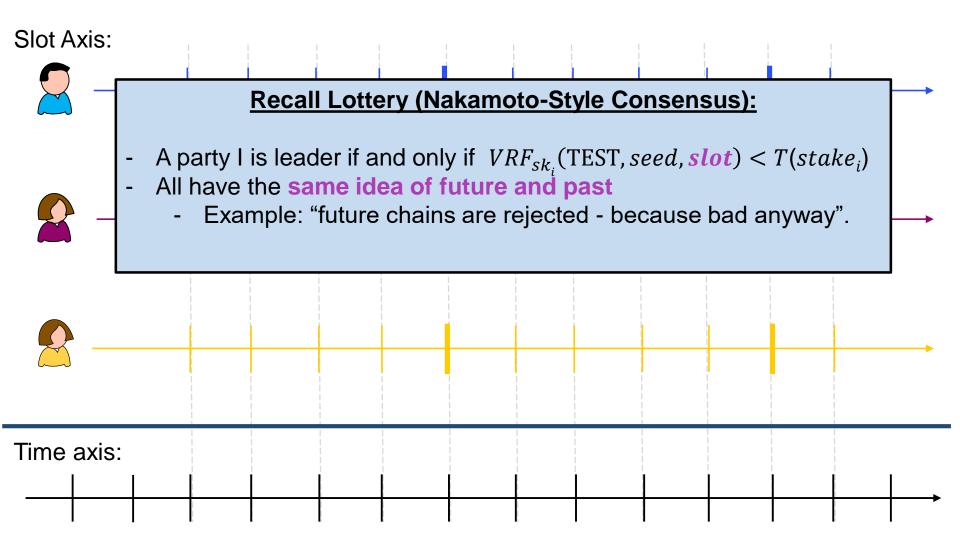












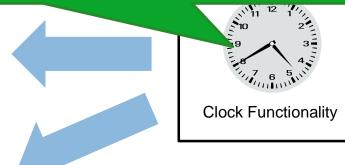
#### So far

#### Bervice

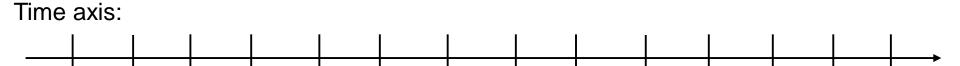
#### **Strong Assumption:**

- Perfect time coordination for everyone, including newly joining parties.
- Needs another protocol eventually, e.g. NTP.

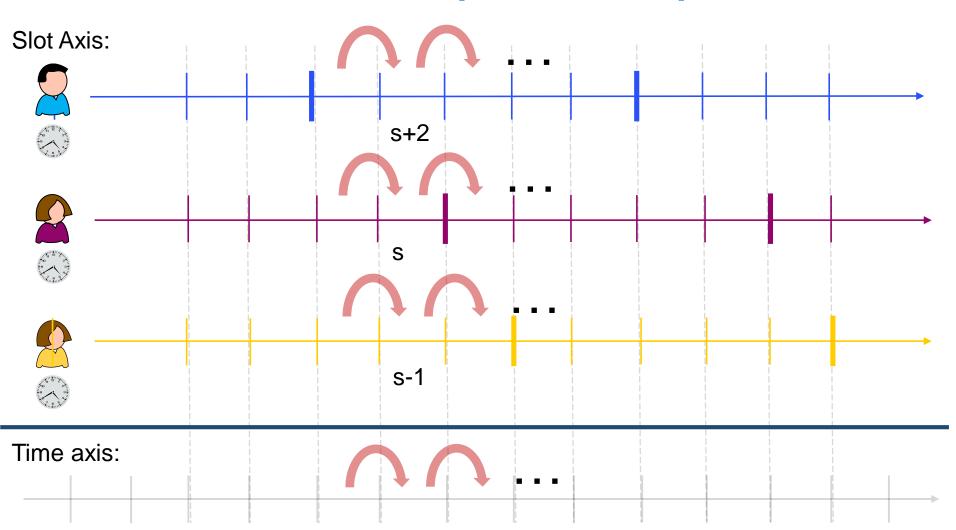




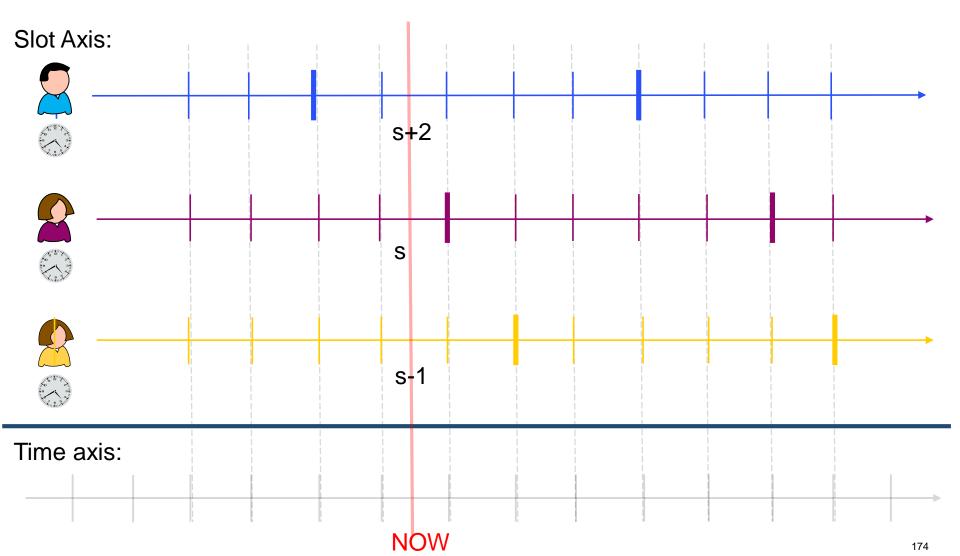




# **Better: Same-Speed Assumption**



# **Coping with Imperfect Coordination**



If we manage to keep all honest parties somewhat close we're kind of good.

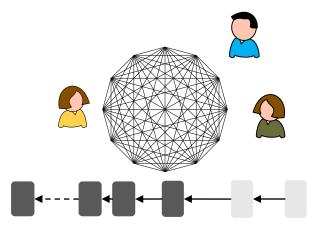
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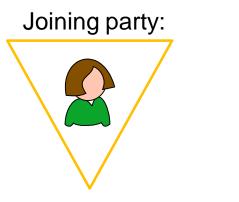
- Close together: Honest parties' timestamps are never more than Δ apart (order of network delay).
- Small adjustments needed to Ouroboros Genesis to deal with future chains

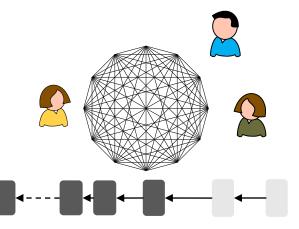
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- Close together: Honest parties' timestamps are never more than Δ apart (order of network delay).
- Small adjustments needed to Ouroboros Genesis to deal with future chains.

- → Same-speed: Initial parties do stay close.
- → Joining parties have a harder life...



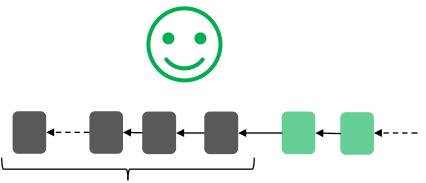








# Joining party:

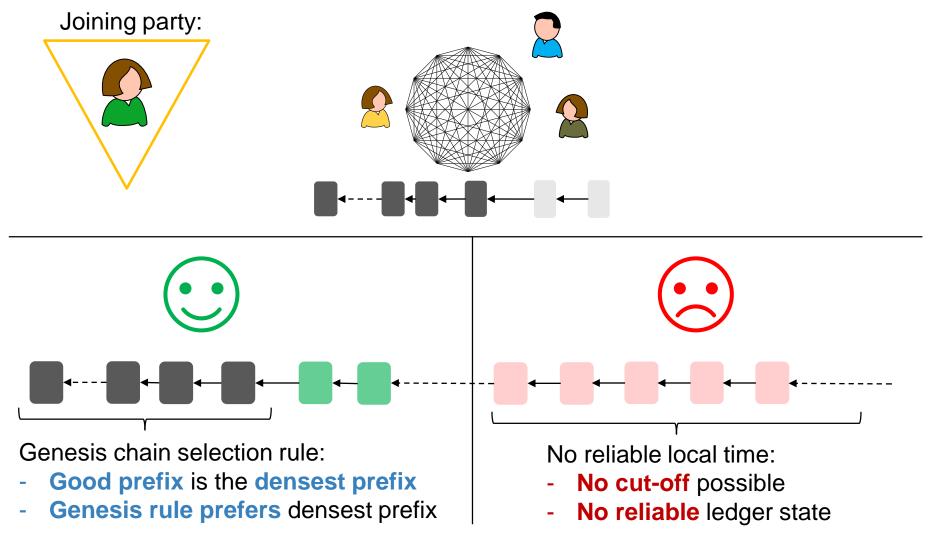




- Good prefix is the densest prefix
- Genesis rule prefers densest prefix



### Genesis: Situation not that bad...



### The Synchronization Problem

- Joining parties: Need to bootstrap a good timestamp
  - Only source of information: network traffic and genesis block.
  - Good: Within the  $\Delta$ -interval of existing honest parties.
  - From before: Good timestamp → Good state.
- Bootstrapping under the same assumptions.
  - Same-speed, honest majority, diffusion network, RO
  - The dynamic availability setting (similar to the Bitcoin setting for fixed difficulty).

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This is what Chronos achieves

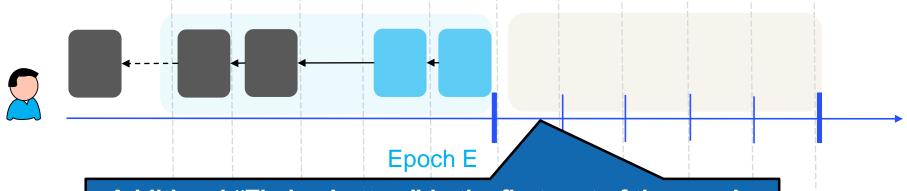
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# **Chronos – Sync-Beacons**



### Additional "Timing Lottery" in the first part of the epoch:

- IF  $VRF_{sk_i}$  ("SYNC", seed, slot)  $< T(stake_i)$  THEN
  - Broadcast Sync-Beacon:

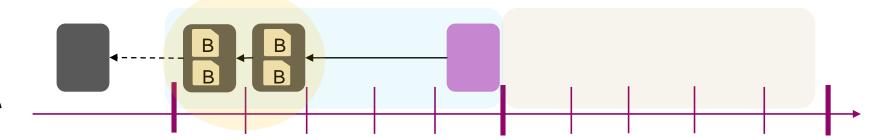


- Normal slot leaders pack transactions + beacons.

- Alert parties: broadcast time-beacons and leave evidence of beacons in the blockchain.
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# **Chronos: Synchronization Procedure**

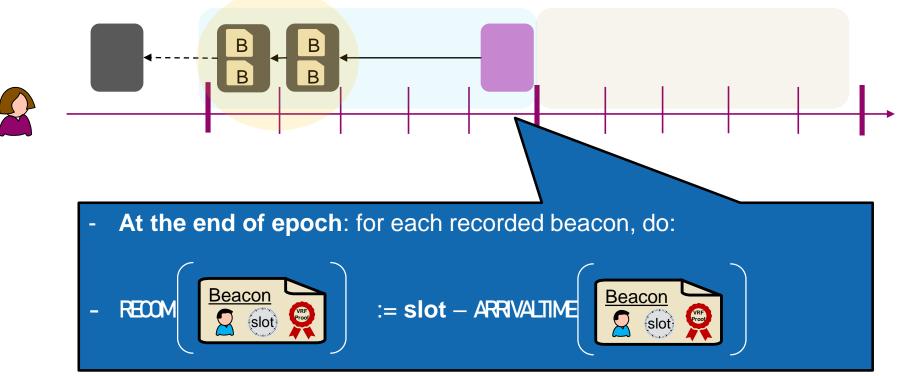
- Throughout the epochs: Alice records the arrival times of valid beacons (filter out duplicates, invalid ones etc.)
- At the end of each epoch: Compute local clock-adjustment.



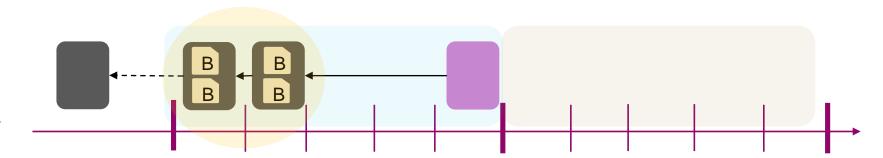


### **Chronos: Synchronization Procedure**

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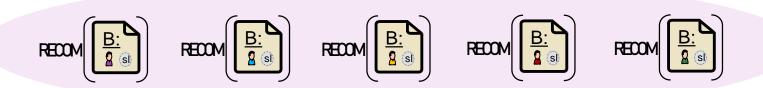
### **Chronos: Synchronization Procedure**

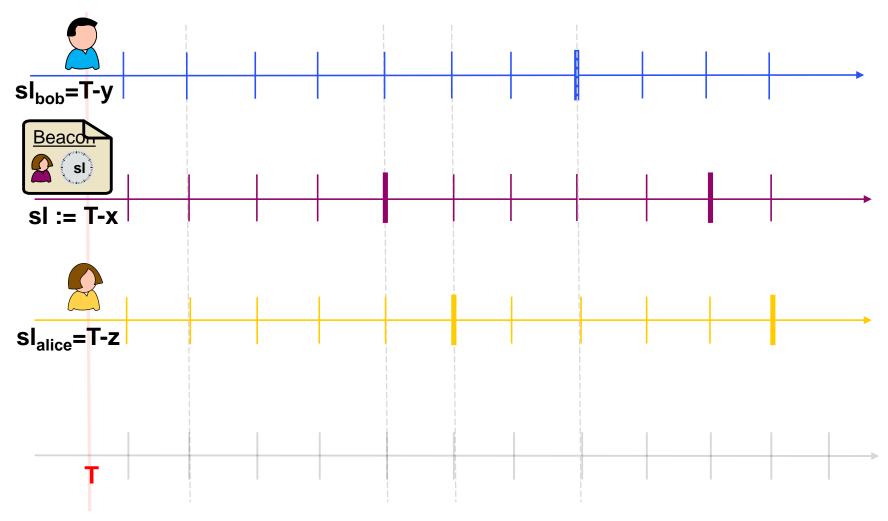


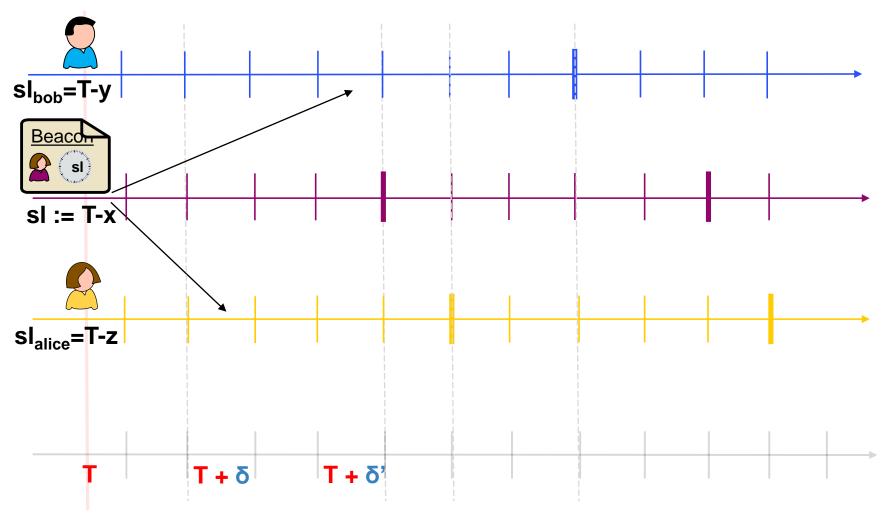


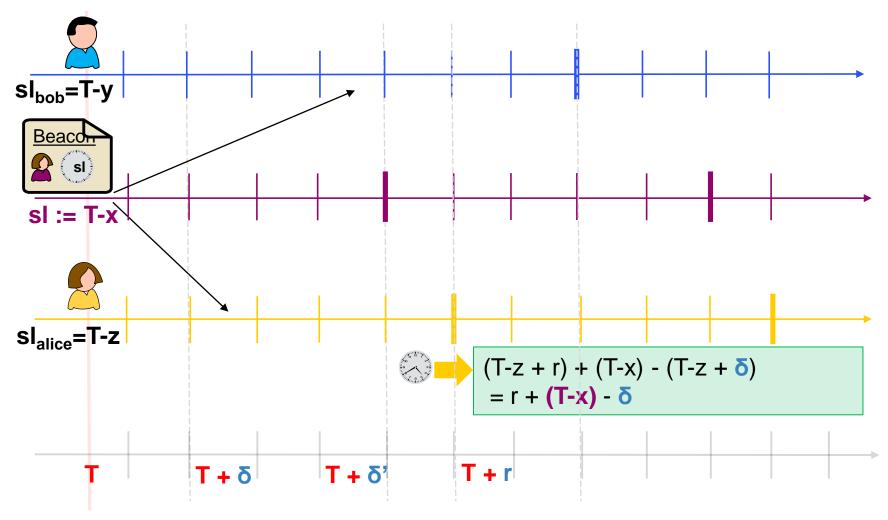
### **Adjustment rule:**

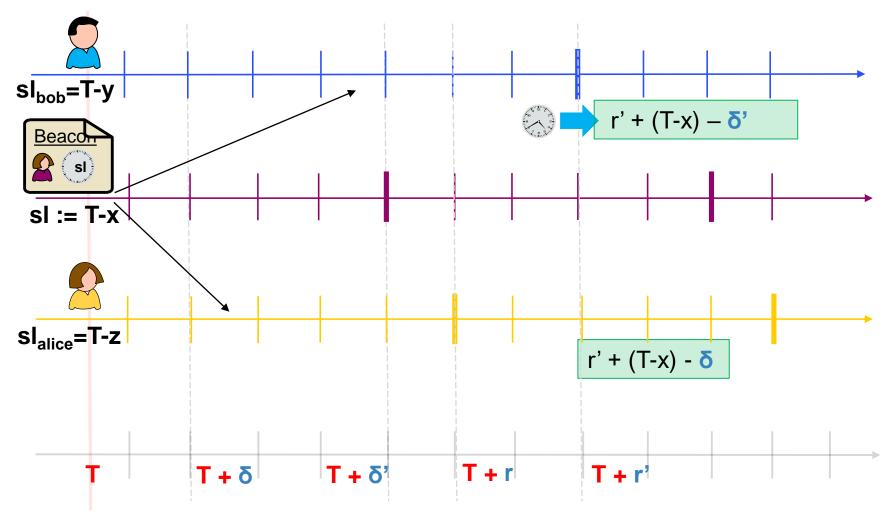
 At the end of epoch: add the median of recommendations to local time:



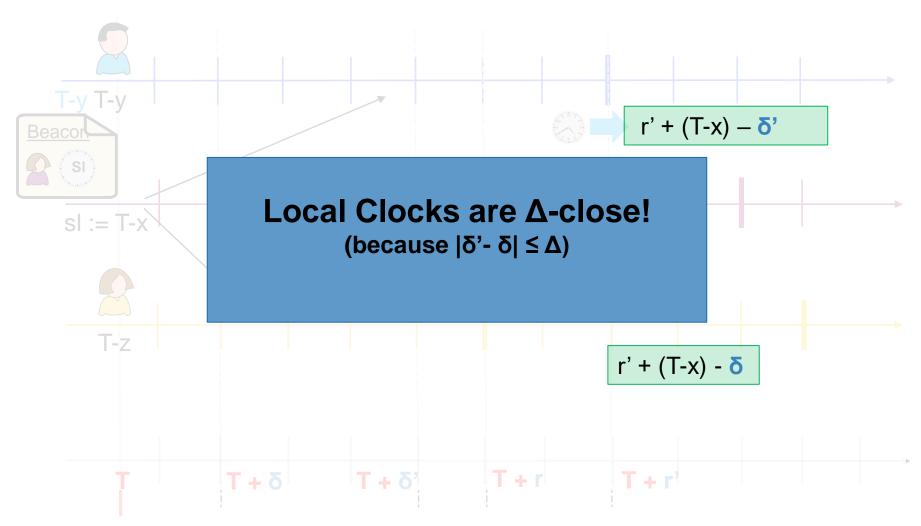






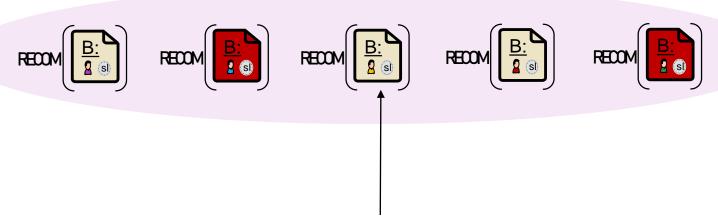


### **Properties of the Synchronization Procedure**



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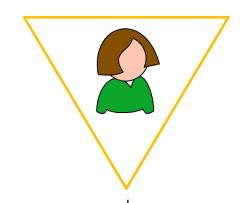




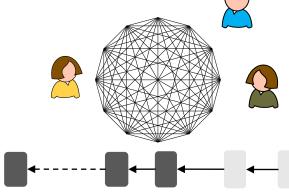
Furthermore, by honest-majority assumption:

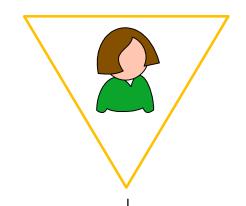
→ Median, i.e., adjustment is bounded.

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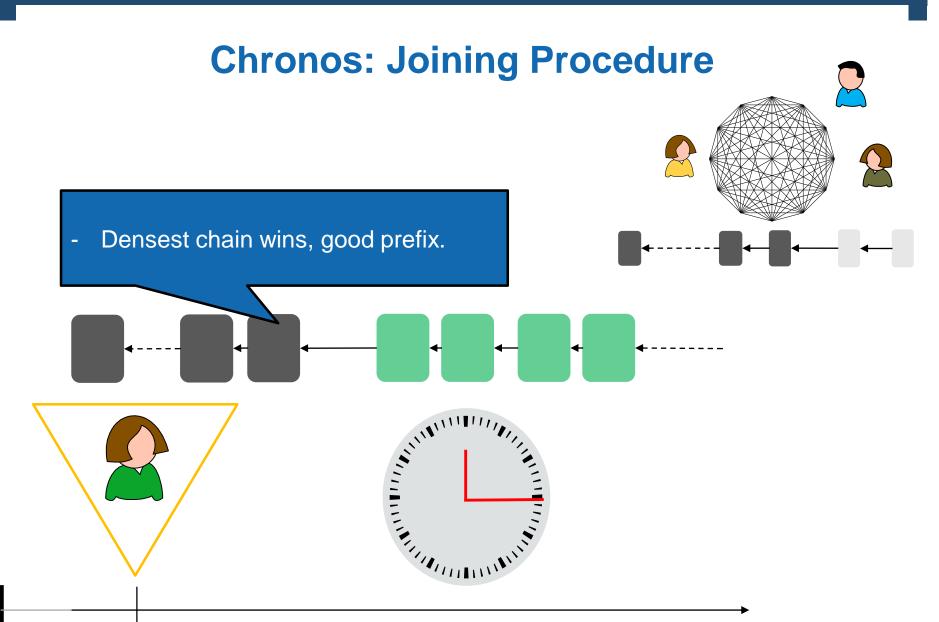


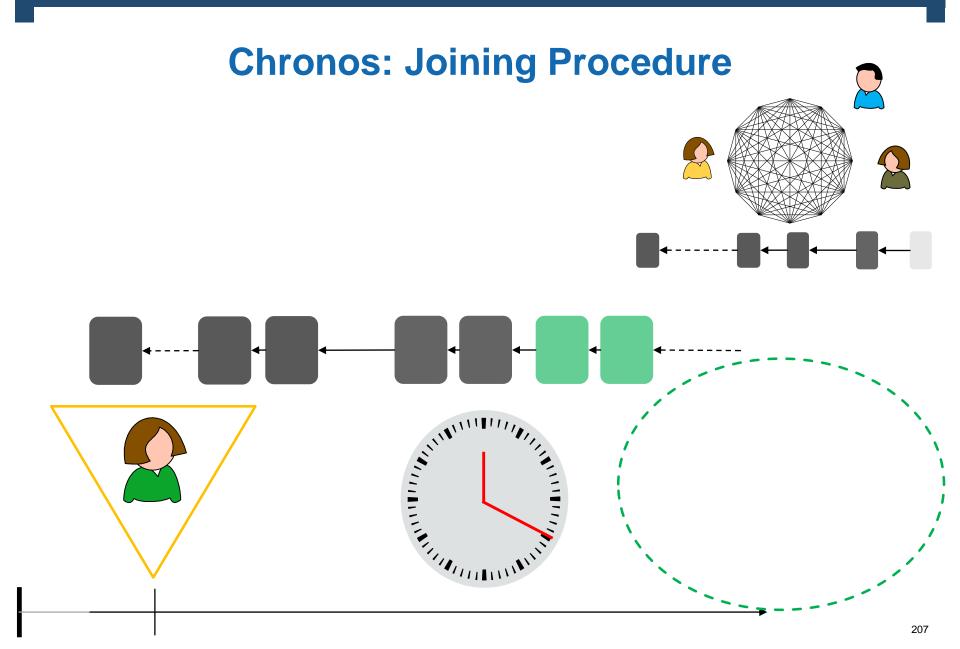


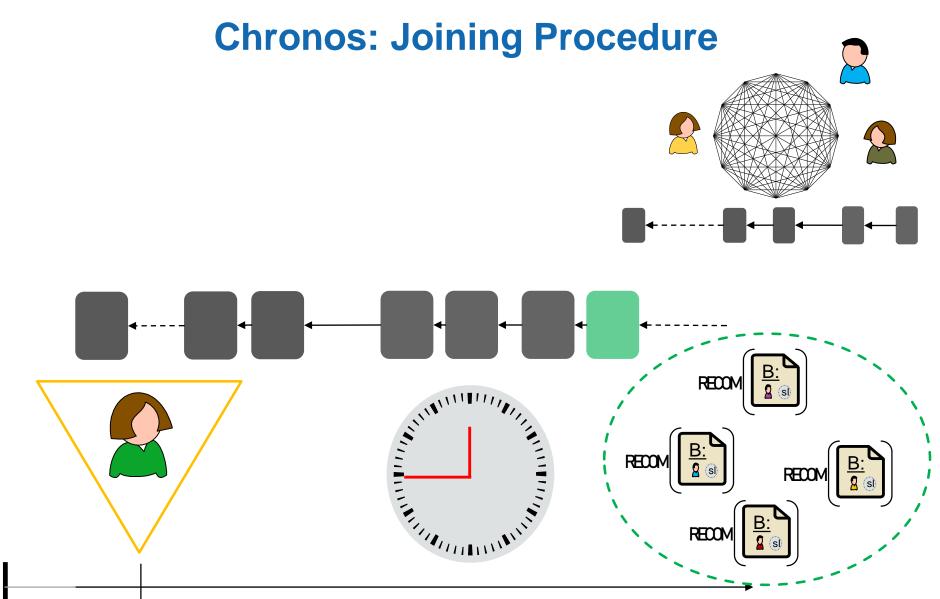


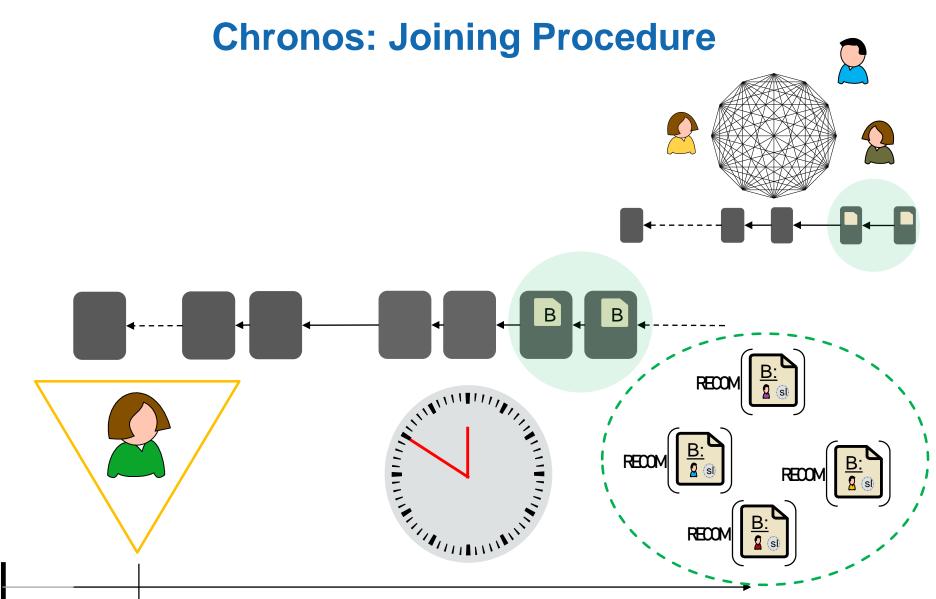






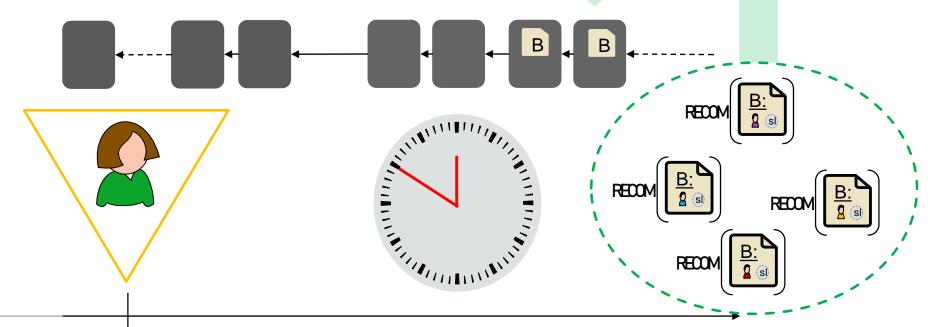




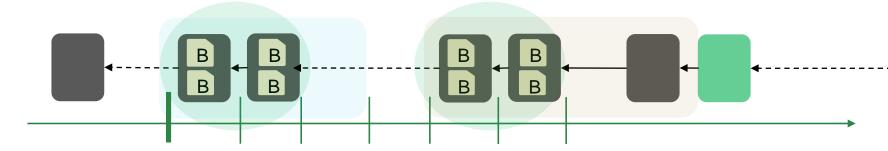


### **Required Beacon Properties:**

- Fresh information: Only generated after becoming online.
- Validated and filtered w.r.t. fresh lottery.
- Contained in common prefix.



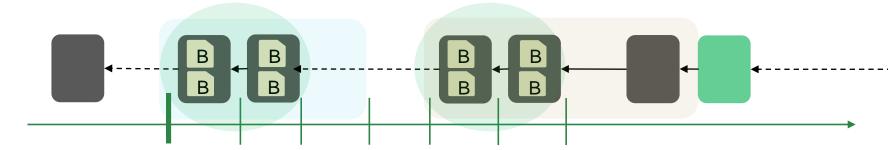
### **Chronos - Summary**





- Bootstrapping the local clock is possible thanks to
  - Agreement on evidence
  - Freshness of beacons: reasoning as before to get Δ-close

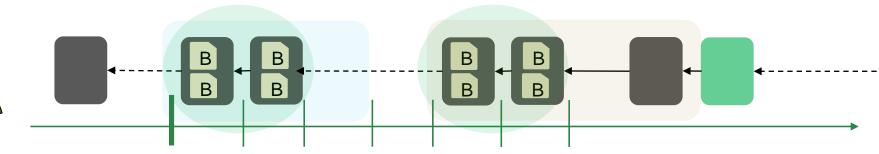
### **Chronos - Summary**





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### **Chronos - Summary**





- Bootstrapping the local clock is possible thanks to:
  - Agreement on evidence
  - Freshness of beacons: reasoning as before to get Δ-close
- Clock adjustments of alert parties can be retraced
  - Stop when computed timestamp is before the next sync-slot.
- Good time-stamp → Good blockchain
  - Cut-off future blocks and the genesis rule guarantees the res

# **Playing With Ouroboros**

Check out the interactive Ouroboros animation:

ouroboros.iohk.io

# **End of the Tutorial – Thank you!**

Ouroboros "Classic" (Crypto 17)

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- + Adaptive Adversaries
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### **Contact information and credits**

Email: christian.badertscher@ed.ac.uk

### References:

Classic: A. Kiayias, A. Russell, B. David, R. Oliynikov: Ouroboros: A Provably Secure Proof-of-

Stake Blockchain Protocol. Crypto 2017

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**Chronos**: C. Badertscher, P. Gaži, A. Kiayias, A. Russell, V. Zikas. Ouroboros Chronos:

Permissionless Clock-Synchronization via Proof-of-Stake. <u>ia.cr/2019/838</u>

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Images: https://openclipart.org/